Philosophy and geometry: theoretical and historical issues

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The best philosophers have been fascinated with geometry ever since the inception of both traditions in 5th century Greece. They have perceived the apparent certainty of geometrical propositions as an intimation that an equally secure knowledge can be had on a subject of the greatest practical concern, viz., the principles of morals. They also have sought to find—as is their wont—reasons for that certainty. This search has given rise to the theoretical and historical issues magisterially reviewed in this book by Lorenzo Magnani. He focuses his attention especially on Kant (Chapters 2 and 3), Poincaré (Chapter 5) and Husserl (Chapter 7.2), and on their original and importantly different ways of explaining the true nature and source of geometrical truth. Despite their diversity, all three can be said to share a trait that is rarely emphasized and which Magnani brings out in full force in his study of Kant. Chapter 2 is called "Geometry: the Model of Knowledge", and his detailed and very able presentation of Kant's philosophy of geometry in this chapter and the next stresses the paradigmatic role of geometrical knowledge in Kant's conception of science. This role is even clearer in the case of Poincaré, whose conventionalism was rooted in his view of geometry but was eventually extended to all the foundations of physics. An enterprising scholar might even try to make a similar point with regard to Husserl, whose views on the establishment of objectivity and science appear to fit geometry better than any other branch of knowledge.

The close relationship between geometry and philosophy in their early days is neatly conveyed by stories about Plato. Not only is there this oft-cited story, telling of the warning at the Academy gate: "Let no one enter who is not versed in geometry", but also there is Plutarch's tale of Plato's arrival in the court of Syracuse: "There was a rush of everyone for philosophy and argumentative discourse (*epi logous*) and the tyrant's palace, they say, was filled with dust by the glut of people doing geometry" (*Dion*, 13.4). This striking episode points to a familiar feature of geometric inquiry that takes pride of place in Magnani's book: the coupling of verbal argument with diagrams, which the Greek drew on sand spread on the floor.

Magnani deals at length with Kant's views on the geometric "construction of concepts", which, as is well-known, he described as a necessary ingredient of geometric proof, by means of which the object that corresponds to a geometric concept is represented "either by imagination alone, in pure intuition, or in accordance therewith also on paper, in empirical intuition—in both cases completely a priori, without having borrowed the pattern from any experience" (Kant, 1781, p. 713; Kant, 1787, p. 741). Magnani explains the link between "construction" in Kant's sense and his famous "schematism", as well as that between both and the "axioms of intuition". This is a notoriously unrewarding task and Magnani does not, in my view, fare much better than the best among his predecessors. To carry clarification any further one must perhaps give too loose a rein to one's fantasy, and we must certainly be grateful to Magnani for not doing so (in stark contrast, say, with Heidegger). He also makes interesting comments on Hintikka's peculiar reading of Kantian "construction", according to which this is none other than the deductive step that the Greeks called *ekthesis*, and is essentially the same as existential instantiation. Until now I had never been able to see a bridge from Hintikka's view of Kant's "construction of concepts in intuition" and Kant's own insistence that intuition is the source of an infinite, a priori given, manifold. In the light of Magnani's exposition I am now inclined to understand this as follows (although I would not dare claim that he shares this view): Unless you have an infinite supply of new names, existential instantiation cannot go on indefinitely; and such a supply may well be equated with—or at any rate likened to—the gift of Kantian intuition (of time?), particularly if new names are generated by adding vertical strokes to the letter x. (Still, it would seem that, for this purpose, a denumerably infinite manifold is enough, whereas Kant's intuitively given manifolds were surely supposed to be continua.)

In Chapters 6 and 7, Magnani develops a more modern and satisfactory approach to geometrical construction as it occurs in real life, and its actual contribution to knowledge. It is impossible to explain this in a short review, but I shall give a few hints. Magnani recalls Peirce's (1931–1958) views on abduction, which the American philosopher portrayed as the mainstay of the advancement of science, but was ready to illustrate with such humble examples as this one: "A man can distinguish different textures of cloth by feeling: but not immediately, for he requires to move fingers over the cloth, which shows that he is obliged to compare sensations of one instant with those of another" (*CP* 5.221). Magnani call this *manipulative abduction*, which "happens when we are thinking *through* doing and not only...about doing" (p. 160). For him, "geometrical construction is a kind of manipulative abduction" (p. 171). In support of this view he twice quotes in full a well-known passage in which Peirce says that geometric reasoning "consists in constructing a diagram according to a general precept, in observing certain relations between parts of that diagram not explicitly required by the precept, showing that these relations will hold for all such diagrams, and in formulating this conclusion in general terms" (*CP* 1.54). It is fair to think of construction, thus envisaged, not as "an art concealed in the depths of the human soul, whose real modes of activity nature is hardly likely ever to allow us to discover" (Kant, 1781, p. 141; 1787, p. 181, speaking of schematism), but as something carried out by hand, on paper, with a pencil, or on a blackboard, with a piece of chalk, or, as by the Syracusan gentry, with a stick on sand. Though non-verbal and even perhaps not properly conceptual, such activity is certainly liable to verbal representation and conceptual analysis. In the book's final section (Chapter 7.6), Magnani refers briefly to recent work on diagrammatic reasoning and so-called heterogeneous deduction, which was pioneered by Barwise and Etchemendy and has been carried further by others. Contrary to all expectations, in this approach "geometrical constructions become rigorous deductive inferences" (p. 208). "The geometrical cases described by Kant, heavy with transcendental meanings, schematism, imagination, synthetic a priori judgment, and so on, are simply deduction, in the formal and modern sense of the term, if plunged into an appropriate logical system endowed with appropriate syntax and semantics" (p. 209).

Since nitpicking is of the essence of scholarly reviews, I shall now indulge in it a little. Magnani writes on p. 78: "Kant did not imagine that non-Euclidean concepts could in some way be constructed in intuition, through the mediation of a *model*, that is preparing and constructing a Euclidean model of a specific non-Euclidean concept (or group of concepts)." Yet Kant wrote that "the use of geometry in natural philosophy would be insecure, unless the notion of space is originally given by the nature of the mind (so that if anyone tries to frame in his mind any relations different from those prescribed by space, he will labor in vain, for he will be compelled to use that very notion in support of his figment)" (Kant, 1770, Section 15E). I find it impossible to make sense of the passage in parentheses unless it refers precisely to the activity of constructing Euclidean models of non-Euclidean geometries (in a broad sense). We now know that one such model (which we ought rather to call quasi-Euclidean, for it would represent plane Lobachevskian geometry on a sphere with radius $\sqrt{-1}$) is mentioned in the *Theorie der Parallellinien* that Kant's fellow Königsbergian Johann Heinrich Lambert (1786) wrote about 1766. There is no evidence that Kant ever saw this tract and the few extant pieces of his correspondence with Lambert do not contain any reference to the subject, but, in the light of the passage I have quoted, it is not unlikely that Kant did hear about it, either from Lambert himself, or from a shared acquaintance, and raised the said objection.

References

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