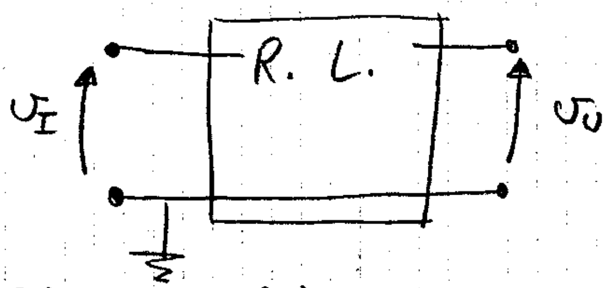
LEZ. 1 - F.d.T - Diagr. Bode

INTRODUZIONE

- Es. su F.d.T e funz. Risp. in freq. di RETI LINEARI (Tracciam. DIAGR. BODE)

- Rete lineare:



F.d.T: $F(s) = \dfrac{v_U(s)}{v_I(s)}$

si ricava dalla rete

- RISPOSTA IN FREQUENZA:

$s \rightarrow j\omega$ → si suppone v_I SINUSOIDALE (→ v_U sinus., stessa ω)

$F(j\omega)$ ← Numero complesso

- DIAGR. di BODE: $|F(j\omega)|_{dB}$ vs. $\log_{10}\left(\frac{\omega}{\omega_0}\right)$

$\measuredangle F(j\omega)$ vs. $\log_{10}\left(\frac{\omega}{\omega_0}\right)$

- POLI e ZERI

$$F(s) = K \cdot s^{\alpha} \cdot \frac{\prod_{1}^{m} i\,(1+s\tau_i)}{\prod_{1}^{n} j\,(1+s\tau_j)}$$

→ esplicitare K reale

α intero

$\tau_j > 0$

→ $(\alpha+n-m)$ ordine di $F(s)$

$z_i = -1/\tau_i \longrightarrow \omega_{zi} = |z_i|$

$p_j = -1/\tau_j \longrightarrow \omega_{pj} = |p_j|$

+ zeri o poli nell'origine

DIAGRAMMI di BODE (APPROX): CONTRIBUTI

! DISEGNARE ASSI !

	VARIAZ. PENDENZA $\|F(j\omega)\|_{dB}$	$\angle F(j\omega)$ per $\omega \to \infty$
POLO	-20 dB/dec	$-90°$
ZERO SX	$+20$	$+90°$
ZERO DX	$+20$	$-90°$
POLI / ZERI NELL'ORIGINE	PARTENZA CON $\omega \to 0$ PEND $= \mp \alpha \cdot 20$ dB/dec	$\angle F(j\omega) = \mp \alpha \cdot 90°$ COSTANTE AL VAR. DI ω

x2

- $\boxed{F(s) = \dfrac{1 - 0.1s}{1 + 0.01s}}$

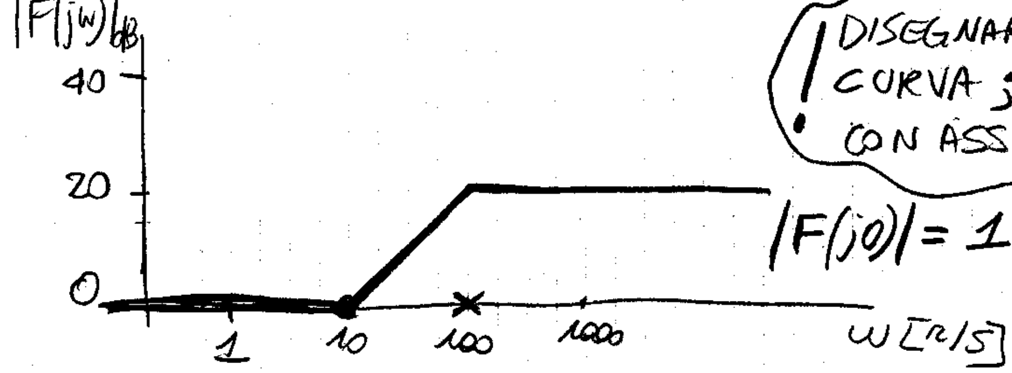
$1 - 0.1s = 0 \longrightarrow z = +10\ r/s$ (ZERO DX) $\rightarrow \omega_z = 10\ r/s$

$1 + 0.01s = 0 \longrightarrow p = -10^2\ r/s \rightarrow \omega_p = 10^2\ r/s$

- $|F(j\omega)|$ vs. ω

  - NO zeri/poli nell'orig.

  ➡ per $\omega \to 0$ pend. $= 0$



! DISEGNARE PRIMA CURVA; POI RIFO CON ASSE TARATO !

$|F(j0)| = 1 = 0\,dB$

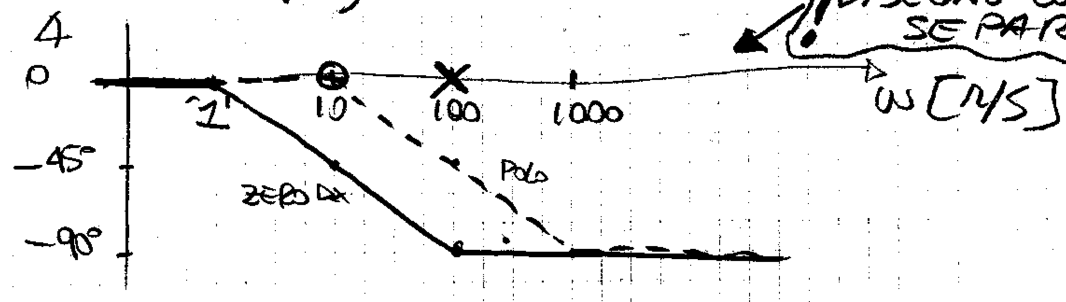
OSSERVAZIONI: - GUAD. UNITARIO x BASSE FREQ

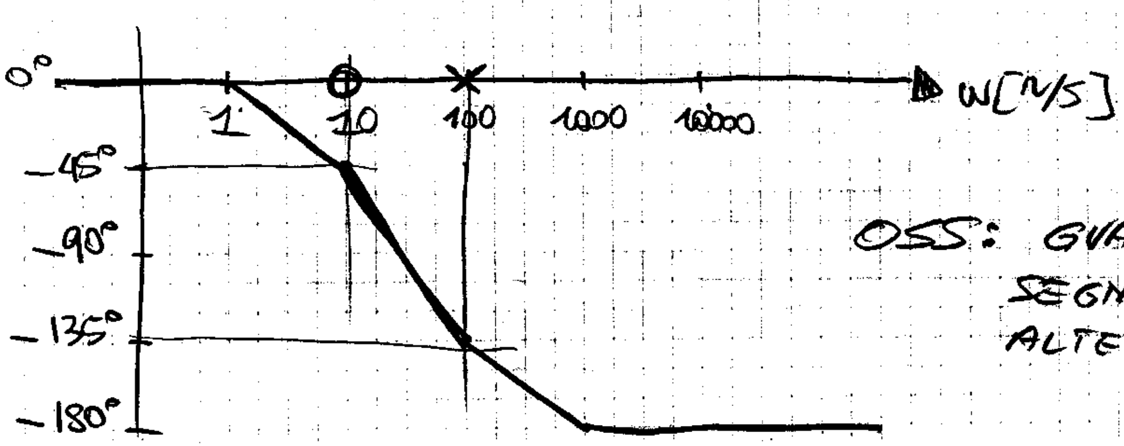
- " $|10|$ " ALTE "

$\downarrow$ IN REALTA' È -10

- $\measuredangle F(j\omega)$ vs. ω

! DISEGNO CONTRIBUTI SEPARATI !





OSS: GUAD. HA SEGNO "-" PER ALTE FREQ.

$$\boxed{F(s) = \frac{10}{s} \cdot \frac{1+s}{1+0.1s}}$$

$z = -1 \ r/s \rightarrow \omega_z = 1 \ r/s$

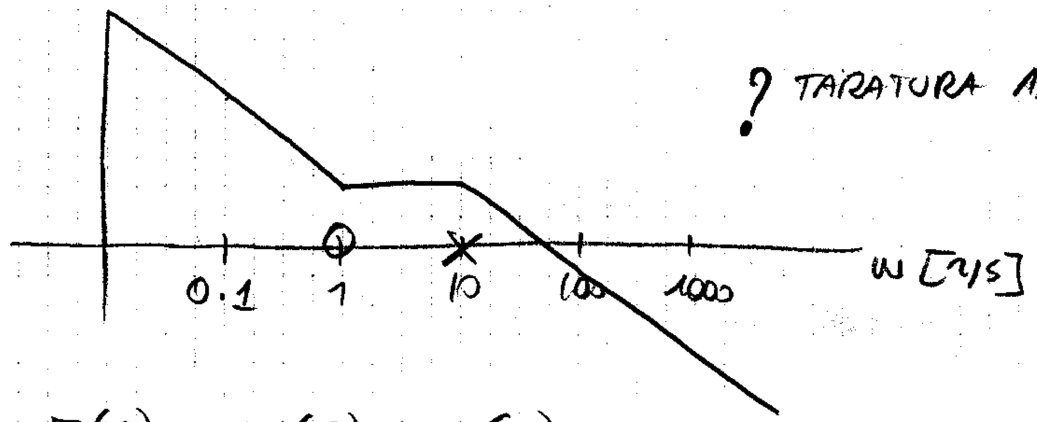
$p_1 = 0 \ r/s \rightarrow \omega_{p1} = 0 \ r/s$ (POLO NELL'ORIG.)

$p_2 = -10 \ r/s \rightarrow \omega_{p2} = 10 \ r/s$

- $|F(j\omega)|$ vs. ω

POLO NELL'ORIG: PER $\omega \rightarrow 0$ PEND. = -20 dB/dec

? TARATURA ASSE ORD ?

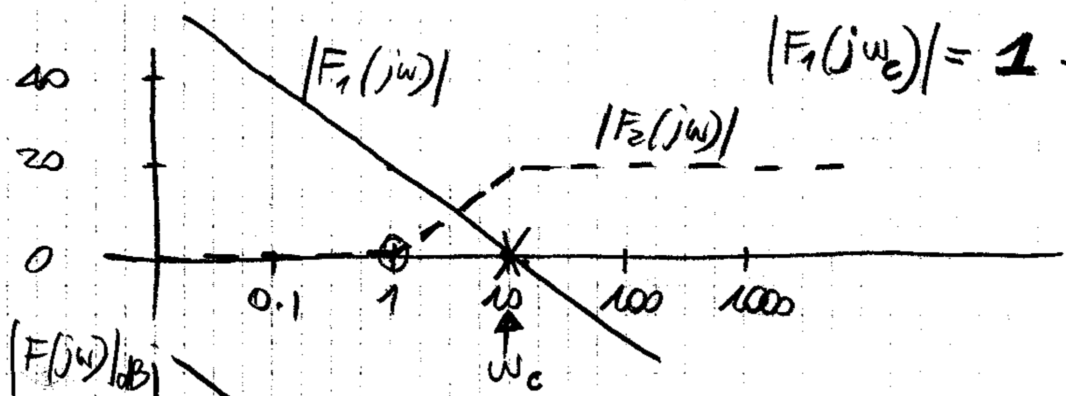


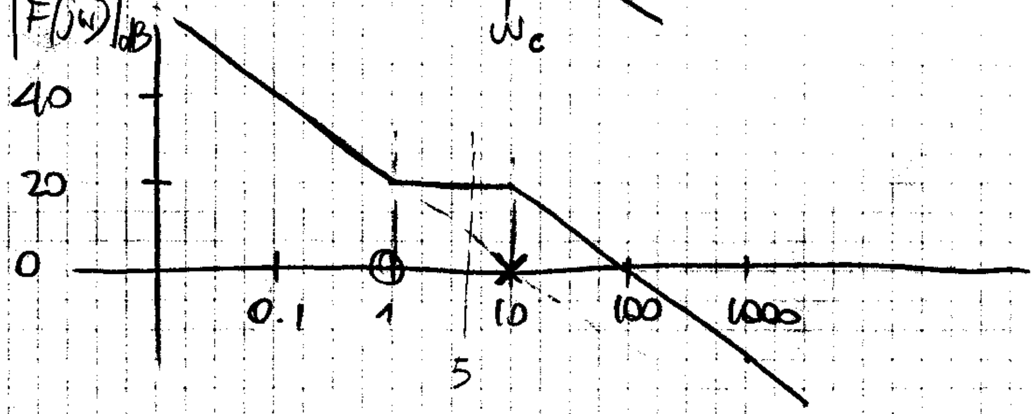
$F(s) = F_1(s) \cdot F_2(s)$

$F_1(s) = \frac{10}{s}$; $F_2(s) = \frac{1+s}{1+0.1s}$

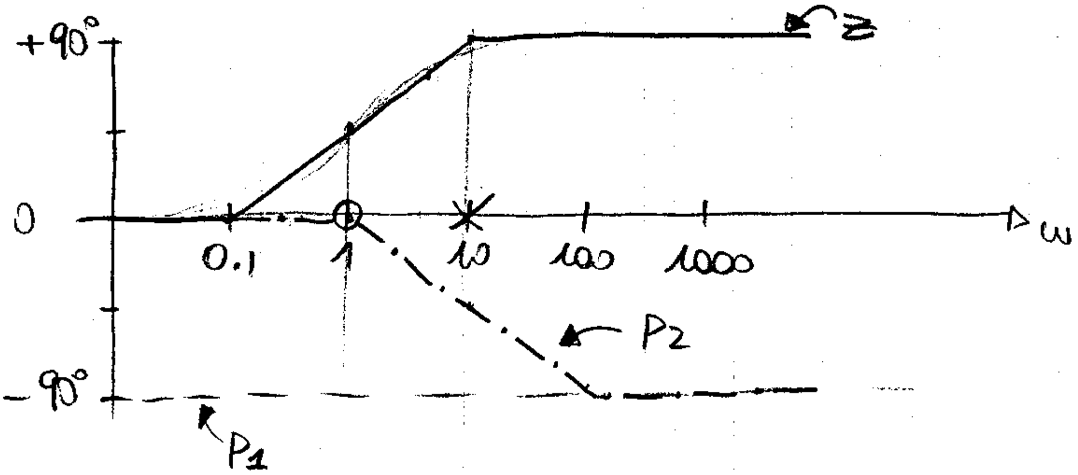
SU DIAGR. BODE : SI SOMMANO :

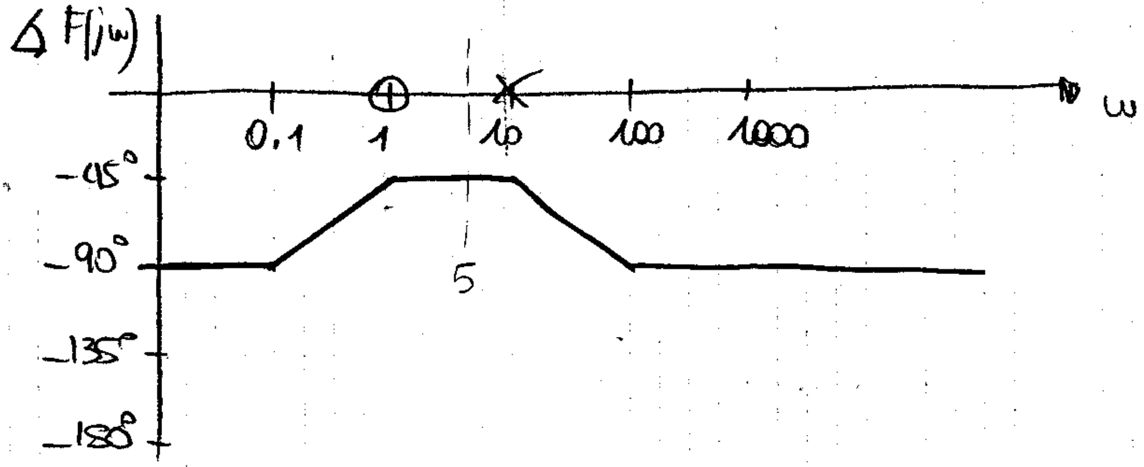
$|F_1(j\omega_c)| = 1 \rightarrow \omega_c = 10 \ r/s$

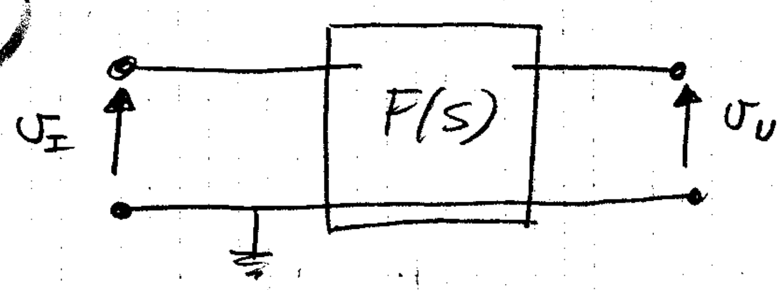




$\measuredangle F(j\omega)$ vs ω







F(s) di prima

$v_I(t) = V_0 \cos(5t)$ $\qquad V_0 = 1\ V$

t in secondi

? $v_U(t)$?

$\omega_s = 5$ r/s

$v_U(t) = V^* \cos(\omega_s t + \varphi)$ perché RETE LIN.

$V^* = V_0 \cdot |F(j\omega_s)|$

$\varphi = \measuredangle F(j\omega_s)$

$$F(j\omega) = \frac{10}{j\omega} \cdot \frac{1+j\omega}{1+0.1\,j\omega}$$

$$|F(j\omega)| = \frac{10}{\omega} \cdot \sqrt{\frac{1+\omega^2}{1+0.01\,\omega^2}}$$

$$|F(j\omega_S)| = \frac{10}{5}\sqrt{\frac{1+25}{1+0.01\cdot 25}} = 2\cdot\sqrt{\frac{26}{1.25}} =$$
$$= 9.12$$

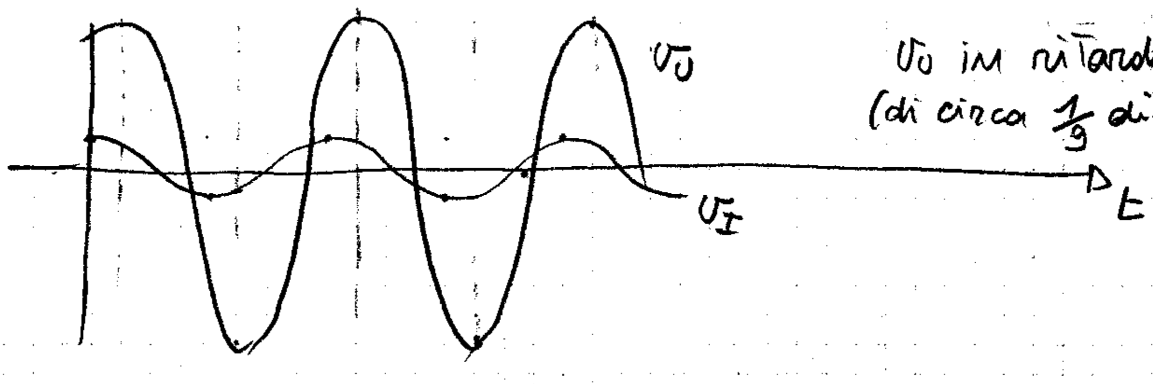
$$V^* = 9.12\ V$$

$$\measuredangle[F(j\omega)] = \measuredangle\left[\frac{10}{j\omega}\right] + \measuredangle[1+j\omega] - \measuredangle[1+0.1j\omega] =$$
$$= \measuredangle\left[-j\frac{10}{\omega}\right] + \measuredangle[1+j\omega] - \measuredangle[1+0.1j\omega] =$$
$$= -90° + \operatorname{atan}(1\cdot\omega) - \operatorname{atan}(0.1\cdot\omega)$$

$\to$ [sec] $\to$ OK DIMENSIONI !!

$$\measuredangle[F(j\omega_S)] = -90° + \operatorname{atan} 5 - \operatorname{atan} 0.5 =$$
$$= -90° + 78.69° - 26.56° = -37.87°$$

$$\varphi = -37.87° = -0.66 \text{ rad}$$



v_U in ritardo (di circa $\frac{1}{9}$ di periodo)

- VERIFICA: $|F(j\omega_S)|_{dB} = 20\cdot\log_{10} 9.12 = +19.2$ dB
CON DIAGR. APPROX.

OK!

!! IN GENERE è sufficiente guardare diagr. BODE !!!

x4

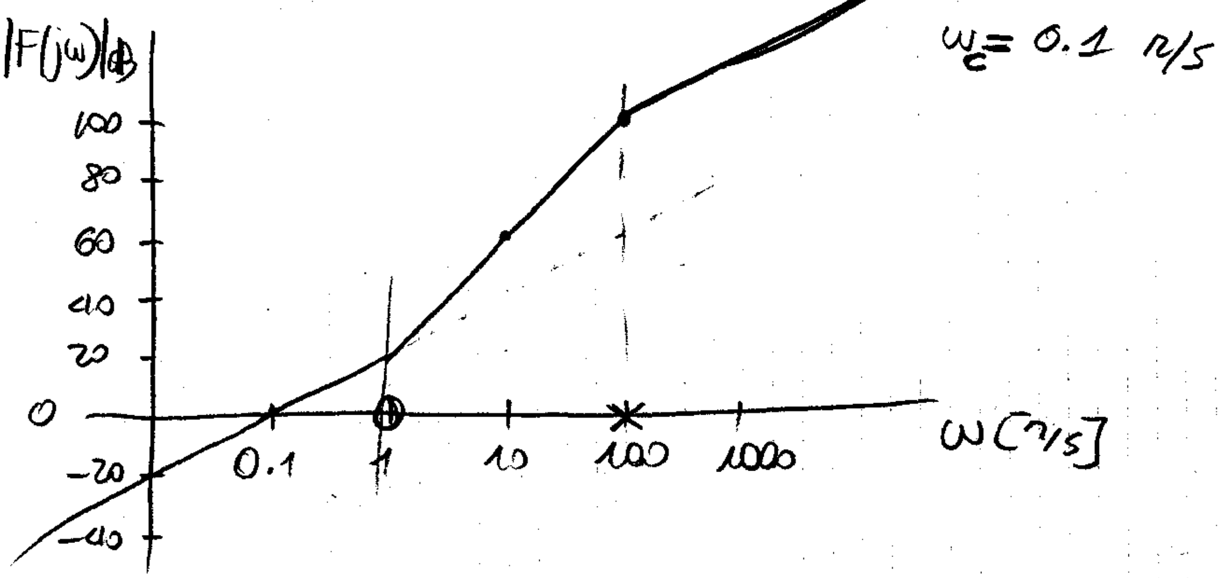
$$\boxed{F(s) = 10\, s \cdot \frac{1+s}{1+0.01s}}$$

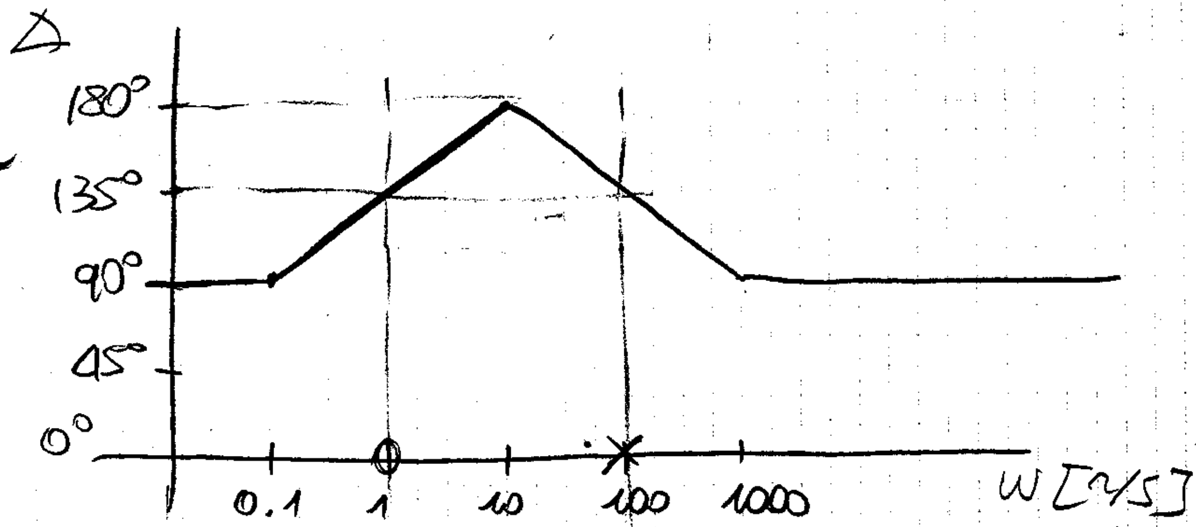
$z_1 = 0$ r/s $\longrightarrow$ $\omega_{z1} = 0$ r/s (zero nell'ORIG)

$z_2 = -1$ r/s $\longrightarrow$ $\omega_{z2} = 1$ r/s

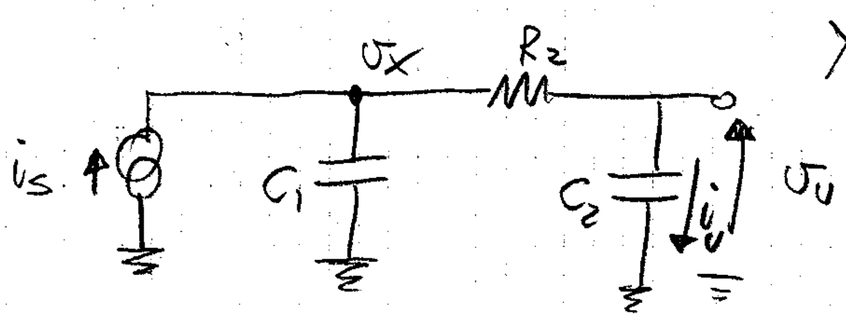
$p_1 = -10^2$ r/s $\longrightarrow$ $\omega_p = 10^2$ r/s

- $|F(j\omega)|$ vs. ω





X



$R_2 = 10\ k\Omega$
$C_1 = 10\ mF$
$C_2 = 1\ \mu F$

① $Z(s) = \dfrac{v_U}{i_S}$ ($[\Omega]$) e
Bode di $F(s) = \dfrac{Z(j\omega)}{R_2}$

② $T(s) = \dfrac{i_U}{i_S}$

$v_X = i_S \cdot Z_{eq}$

$$Z_{eq} = \frac{1}{sC_1} \,//\, \left(R_2 + \frac{1}{sC_2}\right) = \frac{\frac{1}{sC_1}\left(R_2 + \frac{1}{sC_2}\right)}{\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}} =$$

$$= \frac{1}{\cancel{s}\cancel{C_1}} \cdot \frac{1+sC_2R_2}{s\cancel{C_2}} \cdot \frac{\cancel{sC_1C_2}}{C_2 + sC_1C_2R_2 + C_1} =$$

$$= \frac{1+sC_2R_2}{s(C_1+C_2)\left[1+s\frac{C_1C_2}{C_1+C_2}R_2\right]} =$$

!! Dire che voglio forma così $(1+s\tau)$

$$= \frac{1}{s(C_1+C_2)} \cdot \frac{1+sC_2R_2}{1+s\frac{C_1C_2}{C_1+C_2}R_2}$$

$$v_U = v_X \cdot \frac{\frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = i_S \cdot \frac{1}{s(C_1+C_2)} \cdot \frac{\cancel{1+sC_2R_2}}{1+s\frac{C_1C_2}{C_1+C_2}R_2} \cdot \frac{1}{\cancel{1+sC_2R_2}}$$

$$Z(s) = \frac{v_U}{i_S} = \frac{1}{s(C_1+C_2)} \cdot \frac{1}{1+s\frac{C_1C_2}{C_1+C_2}R_2}$$

5

$$F(s) = \frac{Z(s)}{R_2} = \frac{1}{s(C_1+C_2)R_2} \cdot \frac{1}{1+s\frac{C_1C_2}{C_1+C_2}\cdot R_2}$$

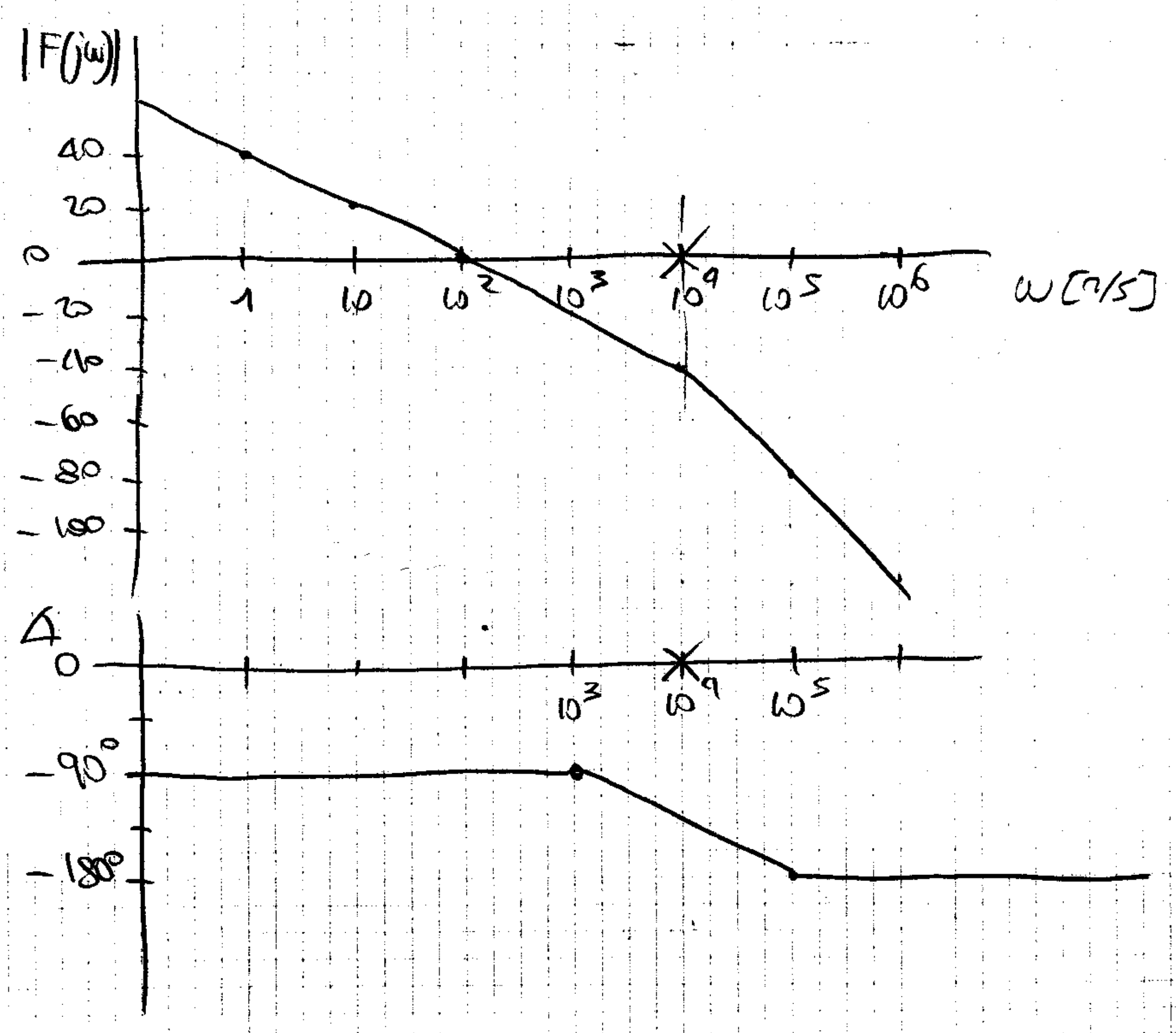
$$C_1 + C_2 \simeq C_2 = 1\mu F$$

$$\frac{C_1C_2}{C_1+C_2} \simeq C_1 = 10 nF$$

$$F(s) = \frac{1}{10^{-6}\cdot 10^4 \cdot s} \cdot \frac{1}{1+s\cdot 10^{-8}\cdot 10^4} =$$

$$F(s) = \frac{1}{0.01 s} \cdot \frac{1}{1+10^{-4}s}$$

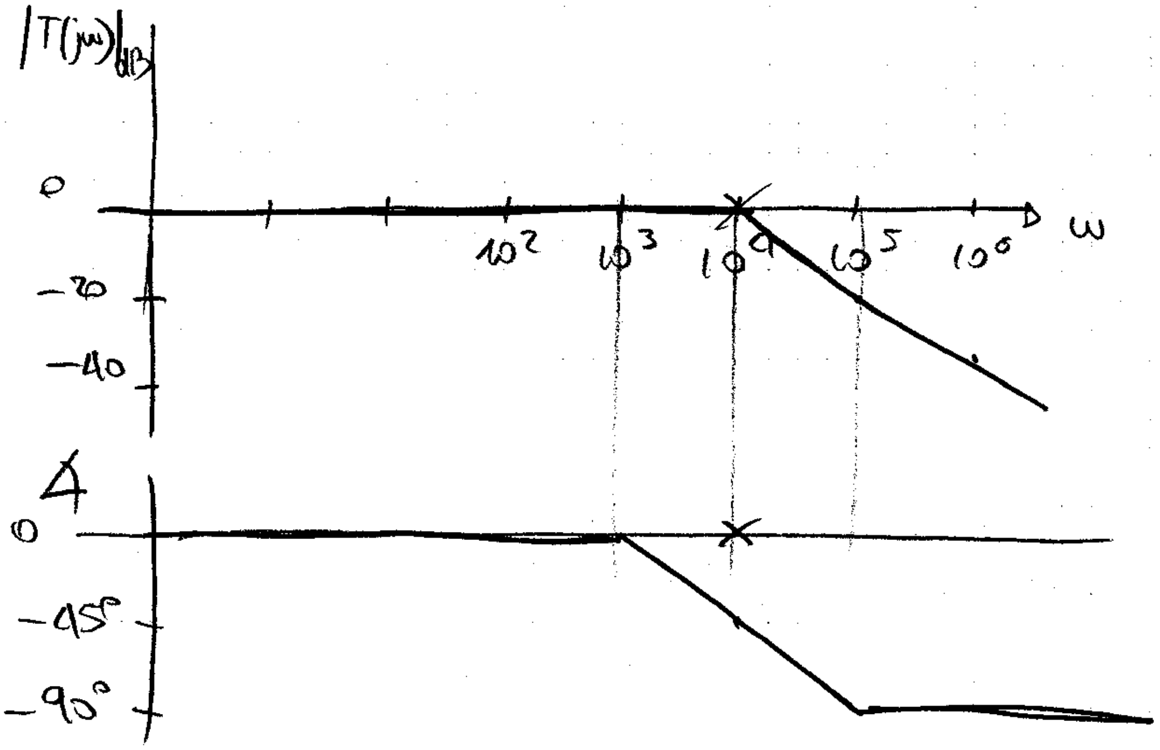
$$P_1 = 0 \qquad P_2 = 10^4 \ r/s$$



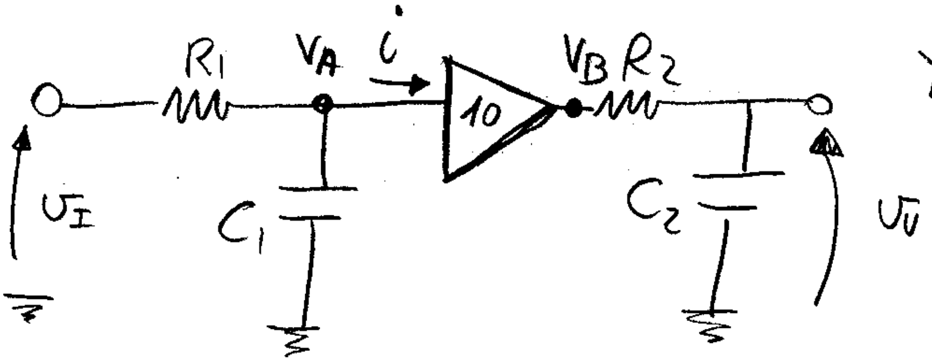
$$i_U = \frac{V_x}{R_2 + \frac{1}{sC_2}} = i_S \cdot \frac{1}{s(C_1+C_2)} \frac{\cancel{1+sC_2R_2}}{1+s\frac{C_1C_2}{C_1+C_2}R_2} \cdot \frac{sC_2}{\cancel{1+sC_2R_2}}$$

$$T(s) = \frac{i_U}{i_S} = \frac{C_2}{C_1+C_2} \cdot \frac{1}{1+s\frac{C_1C_2}{C_1+C_2}R_2}$$

$$T(s) \simeq \frac{1}{1+10^{-4}s}$$



OSSERVAZ: La RISP. IN FREQ. di UNA RETE DIPENDE DALLA GRANDEZZA di USCITA !



X 6

$R_1 = 10\ k\Omega$
$R_2 = 100\ \Omega$
$C_1 = 10\ mF$
$C_2 = 1\ \mu F$

? - $F(s) = \frac{v_U}{v_I}$
- Bode

Amplificatore IDEALE : $i = 0$

$v_B = 10 \cdot v_A$

$$v_A = v_I \cdot \frac{1}{1 + sC_1R_1}$$

$$v_U = v_B \cdot \frac{10}{1 + sC_2R_2}$$

$$F(s) = \frac{v_U}{v_I} = \frac{10}{(1+sC_1R_1)(1+sC_2R_2)}$$

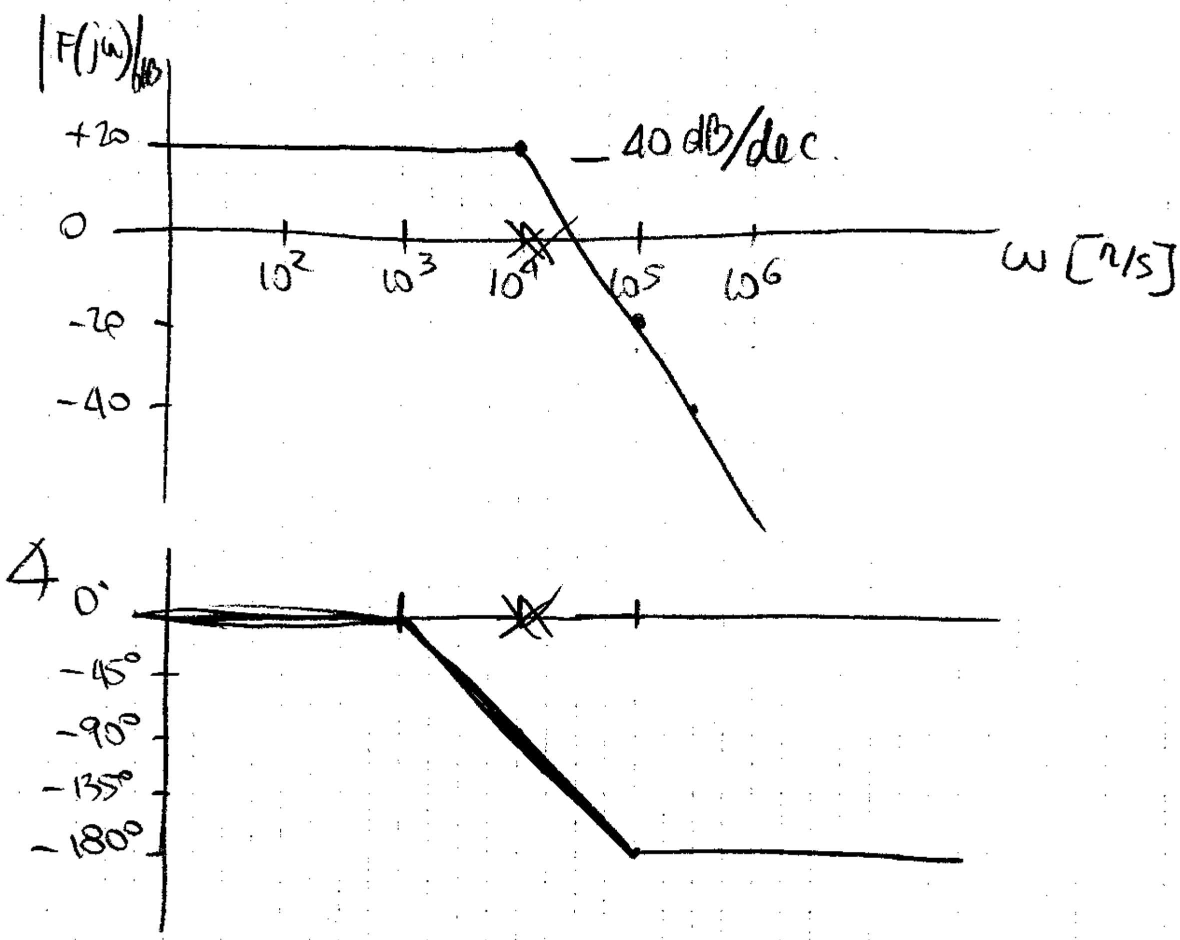
$$C_1R_1 = 10^{-8} \cdot 10^4 = 10^{-4}\ s$$

$$C_2R_2 = 10^{-6} \cdot 10^2 = 10^{-4}\ s$$

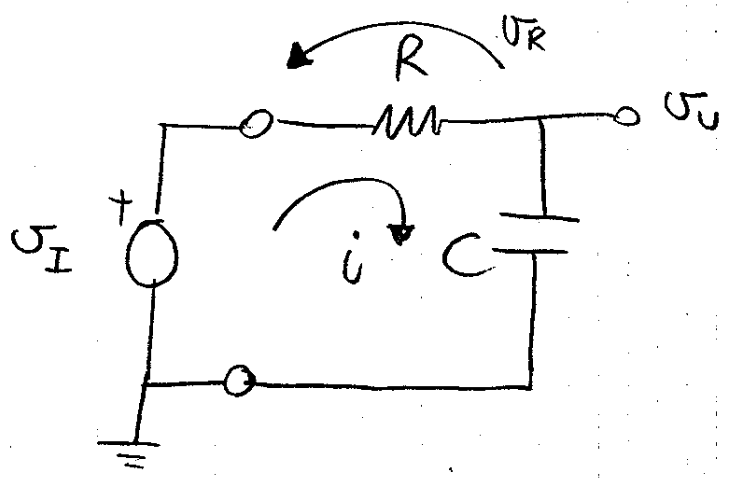
$$F(s) = \frac{10}{(1+10^{-4}s)(1+10^{-4}s)} = \frac{10}{(1+10^{-4}s)^2}$$

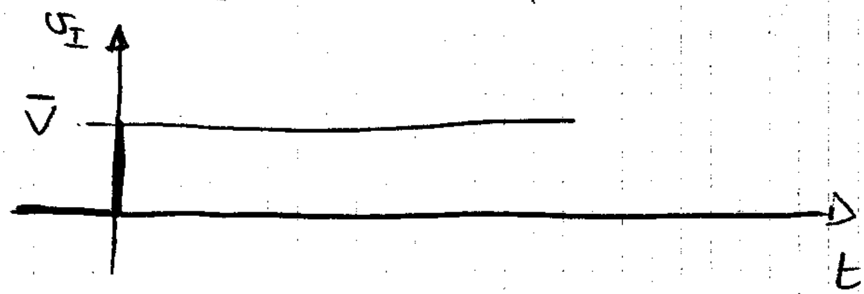
$P_1 = P_2 = -10^4\ r/s \longrightarrow \omega = 10^4\ r/s$

└ <u>POLO DOPPIO</u>


|F(jω)|dB
+20
0
-20
-40
10^2
10^3
10^4
10^5
10^6
_ 40 dB/dec
ω [r/s]
∠
0°
-45°
-90°
-135°
-180°

CIRCUITI RC, RISPOSTA AL GRADINO: SOLUZ. NEL DOMINIO DEL TEMPO





? $v_U(t)$?

$$v_I = v_I(t) \; ; \quad i = i(t) \; ; \quad v_U = v_U(t)$$

$$\begin{cases} i(t) = C \cdot \dfrac{d v_U(t)}{dt} \\ i(t) = \dfrac{v_I(t) - v_U(t)}{R} \end{cases}$$

$$C \frac{d}{dt} v_U(t) = \frac{v_I(t)}{R} - \frac{v_U(t)}{R}$$

$$\tau \frac{d}{dt} v_U(t) + v_U(t) = v_I(t) \; ; \quad \tau = RC$$

→ Eq. diff. lineare, a coeff. cost., del 1° ordine

per $t > 0$: $\begin{cases} \tau \dfrac{d}{dt} v_U(t) + v_U(t) = \bar{V} \\ V_U(0) = 0 \end{cases}$ (Cond. iniz. scarico)

$$v_U(t) = v_{U,0}(t) + v_{U,1}(t)$$

soluz. omogenea — soluz. partic.

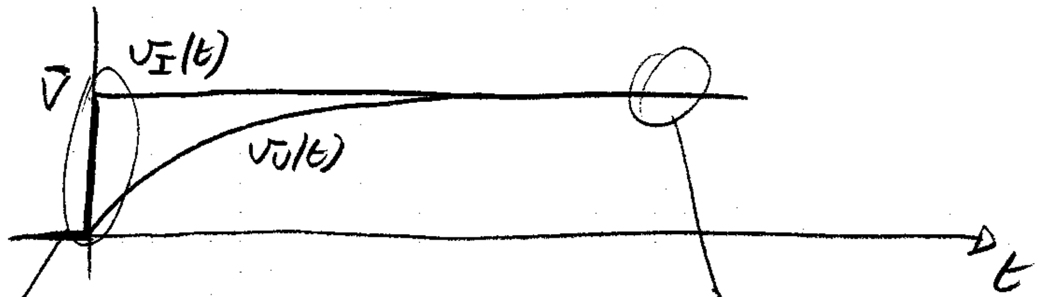
$$v_{U,1}(t) = \bar{V}$$

$$v_{U,0}(t) = V^* e^{-t/\tau}$$ da determ.

$$v_U(t) = \bar{V} + V^* e^{-t/\tau}$$

$$\begin{cases} v_U(0) = 0 \\ \bar{V} + V^* = 0 \end{cases} \rightarrow V^* = -\bar{V}$$

$$v_U(t) = \bar{V} - \bar{V} e^{-t/\tau} = \bar{V}(1 - e^{-t/\tau})$$



PASSA-BASSO

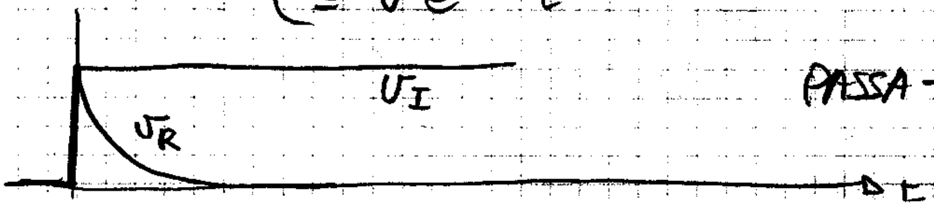
ALTE FREQ (ATTENUATE)

BASSE FREQ (TRASMESSE INDISTURB.)

? TENS. AI CAPI della RESISTENZA ?

$$v_R(t) = v_I(t) - v_U(t)$$

$$v_R(t) = \begin{cases} 0 & t < 0 \\ \bar{V} - \bar{V}(1 - e^{-t/\tau}) = \\ = \bar{V} e^{-t/\tau} & t \geq 0 \end{cases}$$



PASSA-ALTO