Optical and Quantum Electronics **31** *(1999) 367–376*

Semiclassical particle-like description of optical amplifier noise

G. GIULIANI
Dipartimento di Elettronica-Università di Pavia, Via Ferrata 1, I-27100 Pavia, Italy

Received 19 October 1998; accepted 9 November 1998

A semiclassical model of optical amplifier noise in terms of photon-particle description and simple calculations is developed. The active-medium amplifier is modelled as a stochastic photon multiplier, whose statistical properties are derived from well-known results about branching processes. The effects on amplifier output noise caused by random amplification of input photons and by spontaneous emission are treated separately, and it is thus possible to ascribe output noise terms to specific physical mechanisms. In view of this, the 3 dB noise figure limit of optical amplifiers can be entirely ascribed to optical gain randomness. Since the results obtained here are coincident with those from quantum theories, it is concluded that the randomness of optical gain in a particle-like description is the correct semiclassical counterpart of vacuum-field amplification in wave-like formulations.

1. Introduction

In recent years, several treatments of noise in optical amplifiers (OAs) have been developed, either from a quantum-mechanical or semiclassical point of view (see [1], and references therein). Briefly, the rate equation approach [2–5] is based on quantum-mechanical probability transition rules for a two-level atomic medium, and it leads to the Kolmogorov master equations for the photon population which describes the output photon flux in terms of means and higher momenta. The semiclassical beating theory [6] comes to the same results, although it can be shown [7] that the basic assumptions of this theory and the interpretation of noise terms are questionable.

In a number of papers, the statistical theory of *Birth Death Immigration* (BDI) processes [8], first studied by Kendall [9], is applied to optical amplification [10–12]. In these probabilistic approaches, optical amplification is described as the interaction between a stream of particles (photons) and a spatially distributed gain medium capable of multiplying, absorbing or newly generating such particles. A complete statistical description of OA output photons is obtained in terms of probability generating functions (PGF) by means of the Kolmogorov master equations, and results in agreement with quantum-mechanical analyses are attained. However, in these probabilistic treatments the mathematical development is rather involved and the key assumptions leading to correctness of the theory are not evident, nor the amplifier excess noise is ascribed to any specific physical cause.

The aim of the present work is to develop a straightforward semiclassical probabilistic particle-like model with simplified assumptions and analytical calculations, which are based on Kendall's results [9]. This allows to clearly identify the physical mechanisms responsible for OA excess noise, within the frame of the semiclassical corpuscolar description of light.

In a particle-like description of light amplification, the input signal is depicted as a stream of particles (photons) as shown in Fig. 1a. As long as direct detection is considered, the signal-to-noise ratio is calculated as $S/N = \langle n\rangle^2/\langle \Delta n^2\rangle$, where $\langle n\rangle$ is the mean number of photons collected during observation time interval T and $\langle \Delta n^2\rangle$ is the photon number variance. OA noise characteristics are summarised by the noise figure, defined as the ratio of input and output S/N ratios $NF = (S/N)_{\text{in}}/(S/N)_{\text{out}}$.

An *ideal* optical amplifier of power gain G would give as its output a stream of identical photon bunches of the same size G, with each bunch corresponding to an input photon (Fig. 1b). This is the case of a *photon multiplier*, which has unity noise figure. From a quantum-mechanical point of view, ideal photon multiplication (i.e. without excess noise) is not forbidden [13], but the practical implementation of such a device is still to be found.

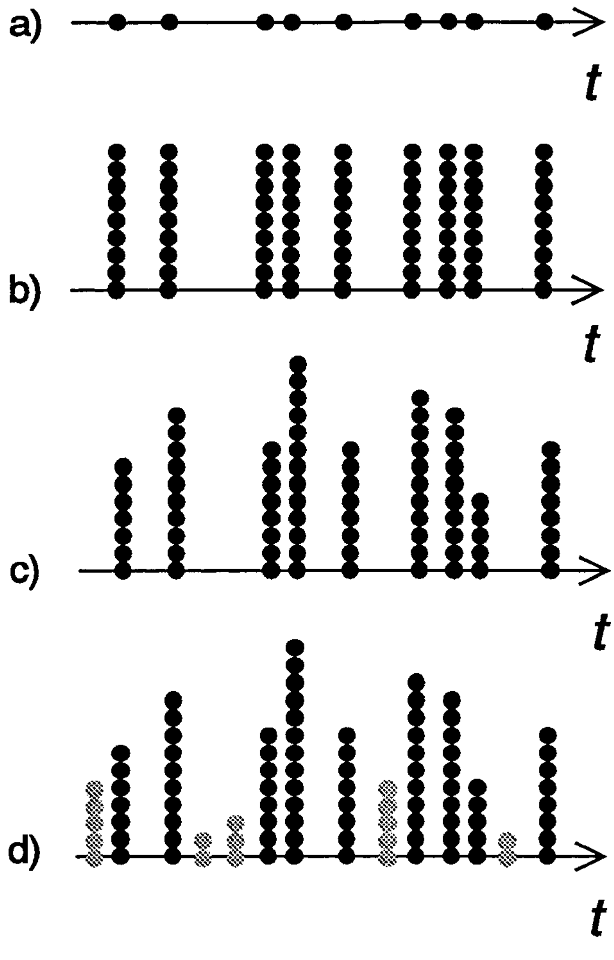

Figure 1 Intuitive representation of photons occurences in the particle-like description. (a) input signal (coherent); (b) output of an *ideal* noiseless optical amplifier (*photon multiplier*); (c) output of an active-medium optical amplifier with random gain (with no spontaneous emission); (d) output of a *real* active-medium optical amplifier with random gain and spontaneous emission.

Optical amplifiers viable for optical communications are *active medium* amplifiers and are based on the principle of stimulated emission which is distributed along the amplifier physical length. Thus, according to the probabilistic nature of stimulated emission phenomena, the number of emissions of new photons stimulated by an original input photon is a stochastic variable, i.e. the optical gain exhibits randomness. The active-medium OA can be regarded as a non-ideal (random) photon multiplier, where the non-ideality lies in the stochastic nature of optical gain. The output of such a random photon multiplier (depicted in Fig. 1c) is made of photon bunches of random size, with the mean size equal to power gain G. In this case the output is more noisy than the input, thus the OA noise figure is larger than unity. Real active medium OAs also generate spontaneous photons that are amplified and give rise to Amplified Spontaneous Emission (ASE). The output of real OAs is similar to the one depicted in Fig. 1d, where new ASE photon bunches (in grey) of random size are generated. The ASE photons obviously yield a CW power term and a noise term, associated with the randomness of the size of ASE bunches.

In this paper the active medium OA with distributed gain is modelled as a random photon multiplier, whose single-photon gain variance is directly deduced from the results of Kendall [9]. The output noise term caused by optical gain randomness is then easily obtained. A simple statistical model for the generation and amplification of spontaneous photons is used to calculate CW and noise terms due to ASE. The results obtained for OA output noise are coincident with those of fully quantum-mechanical treatments [2–5], thus confirming the correctness of the present approach.

In this work, the effects on OA output noise caused by the random amplification of input signal photons are treated separately from those related to spontaneous emission and its amplification. In this way it is possible to ascribe output noise terms to specific physical mechanisms within the frame of the particle-like approach. In particular, the well-known 3 dB minimum noise figure of OAs [1] has to be entirely ascribed to gain randomness, and it is not due to a beating between the amplified signal and ASE (in fact, in a particle-like semiclassical treatment, interference of light can hardly be described).

Note worthily, the same probabilistic theory of Kendall applies to the avalanche photodiode (APD). The APD can be regarded as a random particle-multiplier too, where the randomness stems from the probabilistic nature of the ionisation process. In optimum operating conditions an APD has 3 dB noise figure [14] as well as an OA. This fact is not surprising, since the two devices can be mathematically modelled in the same way within the particle approach.

2. Particle-like theory of optical amplification

Within the frame of a particle description, an optical signal can be modelled as a time-continuous stochastic process and depicted as a sequence of unit-area pulses of shape $f(\tau)$ representing the photon occurrences at random times t_i [15]

$$P_s(t) = h\nu \sum_i f(t - t_i) \tag{1}$$

The above representation agrees with the classical photodetection theory first derived by Mandel [16, 17]. The energy accumulated during the observation time T is

$$E(t,T) = \int_t^{t+T} P_s(t')\,dt' \tag{2}$$

and the number of collected photons is $n_s(t,T) = E(t,T)/(h\nu)$.

All the stochastic processes considered here are supposed to be time-independent. Hence, only time averages of the random variables are needed to derive first- and second-order statistics, and the explicit time dependence used in Equations 1 and 2 can be dropped.

If the signal is a coherent state (i.e., radiation emitted by an ideal laser), then the photon arrival times t_i constitute a Poisson process [2, 15], the distribution of the number of photons collected during the observation time T is Poissonian, the mean count is $\langle n_{\mathrm{s}}\rangle = \langle P_{\mathrm{s}}\rangle T/(hv)$ and the variance is $\langle \Delta n_{\mathrm{s}}^2\rangle = \langle n_{\mathrm{s}}\rangle$. Accordingly, the optical signal power has a variance $\langle \Delta P_{\mathrm{s}}^2\rangle = 2hv\langle P_{\mathrm{s}}\rangle B_{\mathrm{el}}$, where $B_{\mathrm{el}} = 1/(2T)$ is the electrical detection bandwidth.

2.1. Optical amplifier model

The OA is modelled as a two-level active medium of length L with upper and lower level populations represented by the excited-state (N_{e}) and ground-state (N_{a}) atomic densities. The stimulated emission and absorption cross sections are σ_{e} and σ_{a}. A single photon travelling along the OA has a probability per unit length $\gamma = \sigma_{\mathrm{e}}N_{\mathrm{e}}$ of generating another photon by stimulated emission and a probability per unit length $\alpha = \sigma_{\mathrm{a}}N_{\mathrm{a}}$ of being absorbed. For one signal photon entering the OA there is, at the output, a bunch of Γ photons, where Γ is a random variable representing the optical gain experienced by the considered input photon (see Fig. 1). Thus, there is a statistical Birth-Death process (with birth-rate γ and death rate α) along the amplifier, whose complete statistical characterisation has been found by Kendall in a different general context [9]. After [9], Γ has a mean value (the OA gain) $\langle\Gamma\rangle = G = \mathrm{e}^{(\gamma-\alpha)L}$, and a variance

$$\langle \Delta\Gamma^2\rangle = (2N_{\mathrm{sp}} - 1)G(G-1) \tag{3}$$

The factor $N_{\mathrm{sp}} = \gamma/(\gamma-\alpha)$ is a typical parameter of BD processes. In the scientific literature about optical amplifiers it is usually called "spontaneous emission factor" [1], although it is related to stimulated emission and absorption rather than to spontaneous emission itself. The above definition of N_{sp} can be misleading in the particle approach; the definition "inversion population factor" better applies here.

The above expressions are valid for an OA with constant γ and α, i.e. with a spatially uniform population inversion; the generalisation to the non-uniform case is also treated in detail in Ref. [9].

The OA is not only a random photon multiplier, but also emits spontaneous photons that are amplified along the medium and give rise to the output ASE power P_{ASE}, which is a time-continuous time-independent stochastic variable. Since spontaneous emission is statistically independent from the process of signal amplification, these two effects are treated separately.

Let us write the output power of the OA as the stochastic variable

$$P_{\mathrm{out}} = G_{\mathrm{s}}\cdot P_{\mathrm{s}} + P_{\mathrm{ASE}} \tag{4}$$

where G_{s} is a stochastic variable too, whose statistical properties are to be deduced from those of Γ. The mean value and deviation of the output optical power P_{out} are written as:

$$\langle P_{\mathrm{out}}\rangle = \langle G_{\mathrm{s}}\rangle\cdot\langle P_{\mathrm{s}}\rangle + \langle P_{\mathrm{ASE}}\rangle \tag{5}$$

$$\Delta P_{\mathrm{out}} = \Delta G_{\mathrm{s}}\cdot\langle P_{\mathrm{s}}\rangle + \langle G_{\mathrm{s}}\rangle\cdot\Delta P_{\mathrm{s}} + \Delta G_{\mathrm{s}}\cdot\Delta P_{\mathrm{s}} + \Delta P_{\mathrm{ASE}} \tag{6}$$

Since all the fluctuations are uncorrelated with each other, we can find the variance of the output power as

$$\langle \Delta P_{\mathrm{out}}^2 \rangle = \langle \Delta G_{\mathrm{s}}^2 \rangle \cdot \langle P_{\mathrm{s}} \rangle^2 + \langle G_{\mathrm{s}} \rangle^2 \cdot \langle \Delta P_{\mathrm{s}}^2 \rangle + \langle \Delta P_{\mathrm{ASE}}^2 \rangle \tag{7}$$

From (7) it is noticed that output power fluctuations are composed of two contributions. The first arises from amplification of spontaneous emission ($\langle \Delta P_{\mathrm{ASE}}^2 \rangle$), the second arises from statistical amplification of the input signal, namely

$$\langle \Delta P_{\mathrm{out,s}}^2 \rangle = \langle \Delta G_{\mathrm{s}}^2 \rangle \cdot \langle P_{\mathrm{s}} \rangle^2 + \langle G_{\mathrm{s}} \rangle^2 \cdot \langle \Delta P_{\mathrm{s}}^2 \rangle \tag{8}$$

The effects of random amplification of input signal photons and of ASE will be treated, respectively, in Sections 2.2 and 2.3.

2.2. Statistical amplification of the input signal

To determine the noise contribution due to random amplification of the input signal, the variance of G_{s} must be calculated using the statistical properties of Γ. It is

$$G_{\mathrm{s}} = \sum_{i=1}^{n_{\mathrm{s}}} \frac{\Gamma_i}{n_{\mathrm{s}}} \tag{9}$$

where n_{s} is the number of input photons collected during observation time T, and Γ_i are optical gains experienced by each of these photons. According to simple statistical rules [18], the first and second moment of G_{s} are found as: $\langle G_{\mathrm{s}} \rangle = \langle \Gamma \rangle = G; \langle G_{\mathrm{s}}^2 \rangle = \langle \Gamma \rangle^2 + \langle \Delta \Gamma^2 \rangle / \langle n_{\mathrm{s}} \rangle$. Consequently, the variance of G_{s} is

$$\langle \Delta G_{\mathrm{s}}^2 \rangle = \langle G_{\mathrm{s}}^2 \rangle - \langle G_{\mathrm{s}} \rangle^2 = \langle \Delta \Gamma^2 \rangle / \langle n_{\mathrm{s}} \rangle = 2\,h\nu\; B_{\mathrm{el}}(2N_{\mathrm{sp}} - 1) G(G-1) / \langle P_{\mathrm{s}} \rangle \tag{10}$$

The two noise terms in Equation 8 can now be evaluated as follows:

$$\langle G_{\mathrm{s}} \rangle^2 \cdot \langle \Delta P_{\mathrm{s}}^2 \rangle = 2\,h\nu\; B_{\mathrm{el}} G^2 \langle P_{\mathrm{s}} \rangle \tag{11a}$$

$$\langle \Delta G_{\mathrm{s}}^2 \rangle \cdot \langle P_{\mathrm{s}} \rangle^2 = 2\,h\nu\; B_{\mathrm{el}} (2N_{\mathrm{sp}} - 1) G(G-1) \langle P_{\mathrm{s}} \rangle \tag{11b}$$

and the total output fluctuation caused by gain randomness is accordingly

$$\begin{aligned} \langle \Delta P_{\mathrm{out,s}}^2 \rangle &= 2\,h\nu B_{\mathrm{el}} [(2N_{\mathrm{sp}} - 1) G(G-1) + G^2] \langle P_{\mathrm{s}} \rangle \\ &= 2h\nu B_{\mathrm{el}} G \langle P_{\mathrm{s}} \rangle + 4h\nu B_{\mathrm{el}} N_{\mathrm{sp}} (G-1) G \langle P_{\mathrm{s}} \rangle \end{aligned} \tag{12}$$

The above expression represents the power fluctuation at OA output for a Poissonian input signal when spontaneous photons are neglected. This fluctuation is composed of two terms: (i) the amplification of input signal shot-noise (Equation 11a), and (ii) the excess noise introduced by the OA (Equation 11b), whose physical origin is the gain randomness of the active medium.

2.3. Amplification of spontaneous emission

In this section the effects of spontaneous emission are calculated. To this end, the OA is divided into infinitesimal segments of length dz. The power spontaneously emitted by each segment is described as a time-independent time-continuous stochastic process represented by the random variable dP_{sp}. The statistical evolution of spontaneous photons along the remaining part of the OA is calculated as it was done for signal photons in Section 2.2. At

OA output, all the elementary CW and fluctuation contributions (that are uncorrelated with each other) are added together. The OA segmentation is depicted in Fig. 2.

The random variable representing the spontaneously emitted power in each dz can be written as

$$\mathrm{d}P_{\mathrm{sp}} = \langle \mathrm{d}P_{\mathrm{sp}} \rangle + \Delta\,\mathrm{d}P_{\mathrm{sp}} \tag{13}$$

According to the semiclassical theory [19], for a single-polarisation mode the mean value is given by

$$\langle \mathrm{d}P_{\mathrm{sp}} \rangle = h\nu B_{\mathrm{o}} \gamma\,\mathrm{d}z \tag{14}$$

where B_{o} is the optical amplifier bandwidth. The spontaneously emitted photons have a Poissonian statistic [19], hence the variance of the elementary contribution is:

$$\langle \Delta\,\mathrm{d}P_{\mathrm{sp}}^2 \rangle = 2\,h\nu \langle \mathrm{d}P_{\mathrm{sp}} \rangle B_{\mathrm{el}} = 2(h\nu)^2 B_{\mathrm{el}} B_{\mathrm{o}} \gamma\,\mathrm{d}z \tag{15}$$

where B_{el} is the electrical (measurement) bandwidth. As depicted in Fig. 2, the elementary spontaneous power term dP_{sp} emitted at z co-ordinate is the input of an amplifier sub-section of length $L - z$. Thus, the present situation is similar to that analysed in Section 2.2, with the input signal replaced by dP_{sp} and the whole OA by the sub-section extending from co-ordinate z to L. More explicitly, a spontaneous photon emitted at z produces, at OA output, a bunch of Γ_z photons, where Γ_z is the random variable representing the gain experienced by that original photon.

The amplified output power originated by spontaneous emission at z is the stochastic variable $\mathrm{d}P_{\mathrm{sp,out}}(z) = G_z \cdot \mathrm{d}P_{\mathrm{sp}}$, where G_z represents the random gain of the active medium contained between z and L. Therefore we have, in analogy with Equation 8

$$\langle \mathrm{d}P_{\mathrm{sp,out}}(z) \rangle = \langle G_z \rangle \cdot \langle \mathrm{d}P_{\mathrm{sp}} \rangle \tag{16}$$

$$\langle \Delta\,\mathrm{d}P_{\mathrm{sp,out}}^2(z) \rangle = \langle G_z \rangle^2 \cdot \langle \Delta\,\mathrm{d}P_{\mathrm{sp}}^2 \rangle + \langle \Delta G_z^2 \rangle \cdot \langle \mathrm{d}P_{\mathrm{sp}} \rangle^2 \tag{17}$$

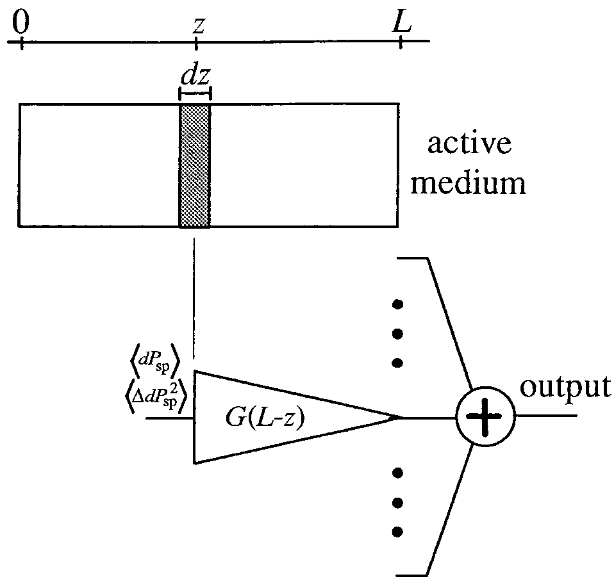

Figure 2 Model of active-medium optical amplifier for the calculation of output ASE power mean and fluctuation. Each spontaneous contribution emitted at co-ordinate z in infinitesimal length dz (with mean $\langle \mathrm{d}P_{\mathrm{sp}} \rangle$ and variance $\langle \Delta \mathrm{d}P_{\mathrm{sp}}^2 \rangle$) is amplified by an amplifier sub-section of length $L - z$ with statistical gain. Amplified contributions (all uncorrelated) are summed at the output.

where $\langle G_z \rangle = G(L-z) = e^{(\gamma-\alpha)(L-z)}$. The variance of the random sub-section gain Γ_z for a single photon is now given by

$$\langle \Delta\Gamma_z^2 \rangle = (2N_{sp} - 1) \cdot G(L-z) \cdot [G(L-z) - 1] \tag{18}$$

According to Equation 10, the variance of G_z is obtained dividing $\langle \Delta\Gamma_z^2 \rangle$ by the mean number of spontaneous photons $\langle n_{sp} \rangle = \langle dP_{sp} \rangle T/hv$ emitted in dz during the observation time T

$$\langle \Delta G_z^2 \rangle = \langle \Delta\Gamma_z^2 \rangle / \langle n_{sp} \rangle = 2hvB_{el}(2N_{sp} - 1) \cdot G(L-z) \cdot [G(L-z) - 1] / \langle dP_{sp} \rangle \tag{19}$$

Using Equation 19, the two terms on the right-hand side of Equation 17 become

$$\langle G_z \rangle^2 \cdot \langle \Delta\, dP_{sp}^2 \rangle = 2(hv)^2 B_{el} B_o \gamma G^2(L-z)\, dz \tag{20a}$$

$$\langle \Delta G_z^2 \rangle \cdot \langle dP_{sp} \rangle^2 = 2(hv)^2 B_{el} B_o \gamma (2N_{sp} - 1) \cdot G(L-z) \cdot [G(L-z) - 1]\, dz \tag{20b}$$

Total output ASE power is the stochastic variable

$$P_{ASE} = \langle P_{ASE} \rangle + \Delta P_{ASE} \tag{21}$$

whose mean and variance are obtained by summing all the uncorrelated infinitesimal contributions (16) and (17). Analytical expressions are calculated by integration over amplifier length L, yielding for the CW power the well-known expression

$$\langle P_{ASE} \rangle = \int_0^L \langle dP_{sp,out}(z) \rangle = N_{sp}(G-1)hvB_o \tag{22}$$

Integration of the variance (Equation 17) yields the following two terms:

$$\begin{aligned} \int_0^L \langle G_z \rangle^2 \cdot \langle \Delta\, dP_{sp}^2 \rangle &= 2(hv)^2 B_{el} B_o \gamma \int_0^L G^2(L-z)\, dz \\ &= (hv)^2 B_{el} B_o N_{sp}(G^2 - 1) = hvB_{el}(G+1)\langle P_{ASE} \rangle \end{aligned} \tag{23a}$$

$$\begin{aligned} \int_0^L \langle \Delta G_z^2 \rangle \cdot \langle dP_{sp} \rangle &= 2(hv)^2 B_{el} B_o (2N_{sp} - 1)\gamma \int_0^L [G^2(L-z) - G(L-z)]\, dz \\ &= (hv)^2 B_{el} B_o N_{sp}(2N_{sp} - 1)(G-1)^2 \\ &= hvB_{el}(2N_{sp} - 1)(G-1)\langle P_{ASE} \rangle \end{aligned} \tag{23b}$$

The total output ASE power fluctuation can thus be obtained as

$$\begin{aligned} \langle \Delta P_{ASE}^2 \rangle &= 2(hv)^2 B_{el} B_o [N_{sp}(G-1) + N_{sp}^2 (G-1)^2] \\ &= 2hvB_{el}\langle P_{ASE} \rangle + 2\langle P_{ASE} \rangle^2 \frac{B_{el}}{B_o} \end{aligned} \tag{24}$$

3. Discussion

From Equations 7, 12 and 24 the total output noise power can be obtained as

$$\langle \Delta P_{out}^2 \rangle = 2hvB_{el}[G\langle P_s \rangle + 2N_{sp}G\langle P_s \rangle (G-1) + \langle P_{ASE} \rangle + \langle P_{ASE} \rangle^2/(hvB_o)] \tag{25}$$

This is a well-known result, coincident with those obtained by the rate-equation approach [3–5], the standard beating theory [6] and probabilistic analyses [10–12].

A peculiarity of the present approach is the possibility of attributing all noise terms to physical causes. This is done as follows:

(i) $2\,h\nu B_{el} G^2\ \langle P_s \rangle$ (Equation 11a) represents the *amplification of the shot-noise carried by the input signal*, which is always present when linear amplification of a shot-noise-limited signal occurs.
(ii) $2\,h\nu B_{el}(2N_{sp}-1)\ G(G-1)\langle P_s \rangle$ (Equation 11b) represents the *amplifier excess noise* caused by optical gain randomness. When $G \gg 1$ and $N_{sp}=1$ this term contributes an amount of noise which equals that of the amplified shot-noise, whence the minimum 3 dB amplifier noise figure.
(iii) $(h\nu)^2 B_{el} B_o N_{sp}(G^2-1)\ = h\nu B_{el}(G+1)\ \langle P_{ASE} \rangle$ (Equation 23a) arises from linear amplification of the shot-noise fluctuations carried by each elementary spontaneous emission term. This overall term can be called *amplification of the shot-noise carried by spontaneous emission*.
(iv) $(h\nu)^2 B_{el} B_o N_{sp}(2N_{sp}-1)(G-1)^2 = h\nu B_{el}(2N_{sp}-1)(G-1)\langle P_{ASE} \rangle$ (Equation 23b) is the excess-noise introduced by each sub-section of the amplifier when elementary spontaneous emission terms are amplified. This overall term can be called *excess noise of spontaneous emission amplification*.

According to the above analysis, output noise is partly attributed to the amplification of *existing* photon-number fluctuation (i.e., input signal shot-noise and elementary spontaneous emission shot-noise) and partly to the *excess noise* added by the random optical amplification process.

It is worth comparing that the so-called beating interpretation [3, 4, 6] states that $2h\nu B_{el} G\langle P_s \rangle$ represents the shot-noise of the amplified signal, $4h\nu B_{el} N_{sp}(G-1)G\langle P_s \rangle$ is the beating between amplified signal and spontaneous emission, $2h\nu B_{el}\langle P_{ASE} \rangle$ is the ASE shot-noise and $2\langle P_{ASE} \rangle^2 (B_{el}/B_o)$ is the ASE-ASE beating. Within the frame of a particle-like description, this interpretation is questionable for two reasons. First, in particle approaches the beating between optical fields cannot be described properly. Second, the term $2h\nu B_{el} G\langle P_s \rangle$ cannot be identified as the shot-noise of the amplified signal. In fact, even an excess-noise-free ideal amplifier (see Fig. 1b) cannot give a shot-noise limited amplified signal, which would imply an improvement in the S/N ratio.

The interpretation of signal-related output fluctuations reported here is consistent with that proposed in Ref. [7], where a rigorous model of optical amplification is formulated in a semiclassical wave-field frame. In Ref. [7] it is found that the dominant OA output noise is due to the amplification of the optical noise carried by the input signal and to the amplification of vacuum-field fluctuations that enter the OA from an idler port. This view is compatible with that of wave-like pure quantum-mechanical treatments [20, 21]. In the present paper it is shown that, within a particle-like description of optical amplification, the effect of gain randomness of a spatially distributed active medium is the correct dual of the amplification of vacuum-field fluctuations in a wave-field approach.

4. Extension to the case of non-Poissonian input signal

To avoid burdening of the notation, all the above expressions were derived with the hypothesis of coherent input state (shot-noise limited signal). The OA output power is a stochastic variable with non-Poissonian statistics, and it is not shot-noise-limited. Thus, the above analysis shall be modified to treat the case of cascaded amplifiers. Here it is demonstrated that the knowledge of the mean and variance of the general input optical signal P_s is sufficient to derive first- and second-order statistics of the OA output.

By introducing the input signal photon number Fano factor

$$F_s = \frac{\langle \Delta n_s^2 \rangle}{\langle n_s \rangle} = \frac{\langle \Delta P_s^2 \rangle}{2h\nu B_{el} \langle P_s \rangle}$$

the result 11b is still valid, while Equation 11a shall be replaced by

$$\langle G \rangle^2 \langle \Delta P_s^2 \rangle = 2h\nu B_{el} G^2 F_s \langle P_s \rangle \tag{26}$$

and consequently Equation 12 becomes

$$\begin{aligned}\langle \Delta P_{out,s}^2 \rangle &= 2h\nu B_{el} \langle P_s \rangle [(2N_{sp} - 1)G(G-1) + F_s G^2] \\ &= 2h\nu B_{el} G \langle P_s \rangle + 4h\nu B_{el} N_{sp}(G-1)G\langle P_s \rangle + G^2(F_s - 1)\end{aligned} \tag{27}$$

As spontaneous emission terms are not affected, the total output power fluctuation can be obtained by summing Equations 27 and 24. The result found here in a straightforward way coincides with that of rate-equations approaches for non-Poissonian input [3–5].

5. Conclusions

In this work, a particle-like semiclassical model of noise in optical amplifier has been presented. The theory is based on the well-known statistical description of multiplicative stochastic variables. Exact expressions for OA output noise have been easily obtained, and a clear physical interpretation of noise terms has been carried out. It is concluded that, within a particle-like description of optical amplification, the effect of gain randomness of a spatially distributed active medium is the correct counterpart of the amplification of vacuum-field fluctuations in a wave-field approach.

Acknowledgements

The author gratefully acknowledges professor Silvano Donati for originating this work with enlightening discussions. This work was supported by FAR-MURST.

References

1. E. DESURVIRE, *Erbium-doped fiber amplifiers: principle and applications* (Wiley, New York, 1994).
2. R. LOUDON, *The quantum theory of light* (Clarendon Press, Oxford, 1983).
3. Y. YAMAMOTO, *IEEE J. Quantum Electron.* **16** (1980) 1073.
4. T. MUKAI, Y. YAMAMOTO and T. KIMURA, *IEEE J. Quantum Electron.* **18** (1982) 1560.
5. R. LOUDON, *IEEE J. Quantum Electron.* **21** (1985) 766.
6. N. A. OLSSON, *J. Lightwave Technol.* **7** (1989) 1071.
7. S. DONATI and G. GIULIANI, *IEEE J. Quantum Electron.* **33** (1997) 1481.
8. W. FELLER, *An introduction to probability theory and its applications*, (Wiley, New York, 1957).
9. D. G. KENDALL, *Ann. Math. Statist.* **19** (1948) 1.
10. P. DIAMENT and M. C. TEICH, *IEEE J. Quantum Electron.* **28** (1992) 1325.
11. T. LI and M. C. TEICH, *IEEE J. Quantum Electron.* **29** (1993) 2568.
12. G. L. CARIOLARO, P. FRANCO, M. MIDRIO and G. L. PIEROBON, *IEEE J. Quantum Electron.* **31** (1995) 1114.
13. G. M. D'ARIANO, *Int. J. Mod. Phys. B* **6** (1992) 1291.
14. R. J. MCINTIRE, *IEEE Trans. Electron Dev.* **13** (1966) 164.
15. D. L. SNYDER and M. I. MILLER, *Random point processes in time and space* (Springer, Berlin, 1991).
16. L. MANDEL, *Proc. Phys. Soc.* **72** (1958) 1037.
17. L. MANDEL, *Proc. Phys. Soc.* **74** (1959) 233.

18. A. VAN DER ZIEL, *Noise. Sources, characterization and measurement* (Prentice-Hall, Englewood cliffs, NJ, 1970).
19. A. YARIV, *Quantum electronics*, 3rd Ed., Wiley (1989).
20. H. A. HAUS and J. A. MULLEN, *Physical Review* **128** (1962) 2407.
21. C. M. CAVES, *Physical Review D* **26** (1982) 1817.