Noise in an Optical Amplifier: Formulation of a New Semiclassical Model

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***Abstract*—After pointing out some questionable assumptions of the standard beating theory, we formulate a new semiclassical wave theory of the noise in optical amplifiers. The theory is simple yet rigorous and uses a few quantum statements in a classical signal framework. The amplifier is modeled as a 2 × 2 port device, and the amplified spontaneous emission and associated noise are shown to be just the amplified coherent state (or vacuum state) fluctuation of the field entering the idler input. The new theory can treat other closely related detection schemes as well, correctly supplying both mean signal and noise.**

***Index Terms*—Amplifier noise, optical amplifiers, optical noise, optical signal detection noise, spontaneous emission.**

I. Introduction

In recent years, noise in optical amplifiers (OA's) has been analyzed by means of several theoretical frameworks [1], each of which has given special emphasis and unveiled a particular aspect of optical amplification. Broadly speaking, these frameworks can be classified as: 1) quantum-mechanical field operator; 2) rate equation; 3) photon multiplication; and 4) field beating; the first two are quantum-mechanical treatments and the others semiclassical. Treatments 1) and 4) consider the wave-like aspect of light and deal with the electric field at optical frequencies, while 2) and 3) consider the particle-like nature of light and deal with photon number.

The field-operator methods [2]–[5] are formal approaches that describe the OA irrespective of the physical mechanism of operation and determine the minimum amount of noise that the OA must add to the amplified signal not to violate quantum-mechanical commutation rules. The well-known result for the minimum noise figure $F_{\min} = 3$ dB is thus obtained.

The rate-equation method [6]–[8], based on the Kolmogorov equation for transition probabilities, considers a real active medium amplifier and allows one to compute the mean value $\langle n_o \rangle$ and variance $\langle \Delta n_o^2 \rangle = \langle n_o^2 \rangle - \langle n_o \rangle^2$ of the number of photons at the OA output as well as the photocount statistics. Explicitly, and with the usual notation, it is

$$\langle n_o \rangle = G\langle n_i \rangle + N_{\rm sp}(G-1) \quad (1)$$

$$\langle \Delta n_o^2 \rangle = G\langle n_i \rangle + N_{\rm sp}(G-1) + 2N_{\rm sp}G(G-1)\langle n_i \rangle + N_{\rm sp}^2(G-1)^2 + G^2(F_i - 1)\langle n_i \rangle \quad (2)$$

where $N_{\rm sp} = \gamma/(\gamma - \alpha)$ is the spontaneous emission factor, $N_{\rm sp}(G-1)$ is the amplified spontaneous emission (ASE) (number of photons per mode at the OA output [1]) and $F_i = \langle \Delta n_i^2 \rangle / \langle n_i \rangle$ is the input-signal Fano factor. By taking into account the optical amplifier bandwidth B_o, the previous results can be expressed in terms of photodetected mean and noise current as

$$\langle I_{\rm ph} \rangle = GI_s + I_{\rm ASE} \quad (3)$$

$$\langle I_{{\rm ph},n}^2 \rangle = [2eGI_s + 2eI_{\rm ASE} + 4GI_sI_{\rm ASE}/B_o + 2I_{\rm ASE}^2/B_o + 2eG^2(F_i - 1)I_s] \cdot B_{el} \quad (4)$$

where $I_s = (e/h\nu)P_s$, with P_s the input signal power, $I_{\rm ASE} = (e/h\nu)P_{\rm ASE}$, with $P_{\rm ASE} = h\nu N_{\rm sp}(G-1)B_o$ the CW ASE power, and $B_{el} \ll B_o$ electrical detection bandwidth.

The above quantum-mechanical treatments are obviously exact but are not versatile (especially the field operator) due to the operator description, which may become a burden in cases of practical interest where realistic fiber links with cascaded OA's are considered. The reduction to a semiclassical description, either particle or wave, is thus a useful schematization, commonly used in many situations occurring in electronics and photonics.

The well-known standard field-beating theory due to Olsson [9], which has been followed by other authors [1], [10], [11], is based on the analysis, at the OA output, of beating terms generated by the square-law process of photodetection of the amplified signal field and the ASE field. The ASE field is postulated to be a Gaussian noise variable with a narrow-band white spectrum of double-sided spectral density $\frac{1}{2}h\nu(G-1)N_{\rm sp}$ and width B_o [in view of quantum-mechanics results; see, for example, (1)]. The beatings between signal and itself, signal and ASE, ASE and itself, correctly give the dc components and the dominant noise term of the photocurrent. However, the exact output noise expression is not obtained from the beatings, since the first two terms of (4) are missing. These missing terms are added heuristically *a posteriori* and attributed to the shot noise of the photodetection process [1], [9], [10]. As will be explained in detail in Section II, the above procedure is not meaningful because the so-called amplified signal and ASE shot noises have no physical significance. Hence, the standard field-beating theory should be regarded as not consistent from a noise-analysis point of view. The standard OA beating theory is derived from an argument that is fairly valid at radio frequencies [12], [13] but fails in describing noise properties of lightwave systems, because at optical frequencies quantum noise predominates over thermal noise and basic quantum principles must be taken into account, even by semiclassical analysis.

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Recently, Berglind and Gillner [14] presented a semiclassical field treatment based on hypotheses differing from Olsson's; namely, they model the OA by means of an electrical equivalent in which the gain is represented by a negative conductance G. Noise sources are explicitly included within the OA by means of a noise spectral density proportional to $h\nu|G|$ associated with the conductance G. The latter assumption is an indirect consequence of the existence of quantum vacuum fluctuations, as clearly explained in [15]. The correct result (4) is attained, and the terms $2eGI_s, 2eI_{\rm ASE}$ are obtained by the calculation, although again attributed to a shot-noise effect. The one proposed in [14] is an elegant approach; however, its formal correctness is hindered by the attempt to conform to the "standard beating" framework by means of a misleading hypothesis on noise sources, as it will be explained in Section II.

In this paper, we present a new semiclassical wave-like description of the OA capable of yielding the correct result (4) for the output noise within a self-consistent frame, which is also valid in general cases (i.e., for coherent photodetection and cascaded amplifiers). Our model is based on a few assumptions from quantum theory. The most relevant assumption concerns vacuum fluctuations which must be taken into account for both the conventional and idler input of the OA and are shown to be the cause of the ASE CW power and added noise. In our treatment, proper electric field fluctuations are superposed to the input signal, contrarily to what it is done in previous semiclassical works [9], [14] that consider a noise-free pure sine wave as input signal to the OA.

It should be observed that a well-proven semiclassical approach is of extreme importance in dealing with engineering applications of optical amplification, expecially if the model is simple and has general validity. We would like to point out that, to the best of our knowledge, no fully rigorous development of a wave-like semiclassical model for photodetection of amplified signals has been performed to date. The method presented in this paper parallels the procedure of the quantum-field approach [3], [5], bearing in mind that we suggest a treatment in which the electric field, and not the corresponding operator, is represented directly, thus allowing a great simplification due to the use of the conventional electrical communication-theory principles and theorems.

The paper is organized as follows. In Section II, we summarize and comment on the standard field-beating theory [9] and on the recent analysis of [14]; in Section III, we present in detail the new semiclassical formulation which overcomes the revealed inconsistencies and fully accounts for the dc as well as the noise components of the OA output; in Section IV, we discuss the obtained results and compare our treatment to others.

II. The Standard Semiclassical Beating Theory

In the commonly accepted derivation of this theory, due to Olsson and Henry [9], the analysis is carried out by looking at the OA output with a photodetector. Here, the average output current is the sum of amplified signal and ASE

$$\langle I_{\rm ph}\rangle = G\langle I_s\rangle + \langle I_{\rm ASE}\rangle \tag{5}$$

and the currents are $(e/h\nu)$ times the powers (assuming unitary photodetector quantum efficiency $\eta = 1$), i.e.,

$$I_s = (e/h\nu)P_s, \quad I_{\rm ASE} = (e/h\nu)P_{\rm ASE}. \tag{6}$$

The electric field amplitudes of signal and ASE at the AO output are written as $E = \sqrt{(2P)}$ (the missing factor Z_0/A is uninfluential). The amplified signal at frequency ν_s is therefore

$$E_s = \sqrt{(2GP_s)}\cos 2\pi\nu_s t. \tag{7}$$

The assumption taken from quantum theory concerns the ASE power and its optical spectral density. For one mode, the CW ASE power is

$$P_{\rm ASE} = h\nu N_{\rm sp}(G-1)B_o \tag{8}$$

where B_o is the amplifier optical bandwidth. The ASE power is assumed uniformly distributed over the optical bandwidth B_o and is decomposed in a number $2M = B_o/\delta\nu$ of elemental intervals $\delta\nu$, each having a power $\delta P_{\rm ASE} = N_{\rm sp}(G-1)h\nu\delta\nu$ and a random phase ϕ_k, uncorrelated for each k. The ASE field is accordingly written as

$$\begin{aligned} E_{\rm ASE} &= \sqrt{2\delta P_{\rm ASE}} \sum_{k=-M,+M} \cos[2\pi(\nu_s + k\delta\nu)t + \phi_k] \\ &= \sqrt{2N_{\rm sp}(G-1)h\nu\delta\nu} \sum_{k=-M,+M} \\ &\quad \cdot \cos[2\pi(\nu_s + k\delta\nu)t + \phi_k]. \end{aligned} \tag{9}$$

In the limit $\delta\nu \to 0$, the above ASE field has the same statistical properties of a Gaussian-distributed narrow-band noise variable [12]. The instantaneous (mean plus fluctuation) photodetected current is then taken as

$$I_{\rm ph}(t) = (e/h\nu)\langle (E_s + E_{\rm ASE})^2\rangle \tag{10}$$

where the average is over optical frequencies. Inserting (7) and (9) in (10) gives explicitly (11), shown at the bottom of the next page. At the right hand side of (11), the first term is the signal–signal beating; the second term is the so-called signal–ASE beating and the third is the ASE–ASE beating. By developing the cosine products in sum and difference of the arguments, and dropping the high-frequency components, the following dc and noise terms are obtained [9]:

1) from the signal–signal beating, the signal dc term GP_s;
2) from the ASE–ASE beating, the ASE dc term $P_{\rm ASE} = h\nu N_{\rm sp}(G-1)B_o$ and a noise term with single-sided spectral density given by

$$S_{I{\rm ph,ASE\text{-}ASE}}(f) = 2I_{\rm ASE}^2/B_o \cdot (1 - f/B_o) \cdot {\rm rect}[0, B_o] \tag{12}$$

where ${\rm rect}[0, f_0] = 1$ for $0 < f < f_0$ and ${\rm rect}[0, f_0] = 0$ for $f < 0$ and $f > f_0$;

3) from the signal–ASE beating, a noise term with single-sided spectral density

$$S_{I{\rm ph},s\text{-}{\rm ASE}}(f) = 4GI_sI_{\rm ASE}/B_o \cdot {\rm rect}[0, B_o/2]. \tag{13}$$

The two above noise terms alone do not give the correct result (4) as obtained by the exact quantum-mechanical approach. To match the correct expression, it is assumed that two white-noise terms $2eGI_s, 2eI_{\rm ASE}$, identified as amplified signal and ASE photocurrent shot noise, shall be added to those obtained by the calculation. The above derivation includes a few inconsistencies on which we will now comment. It is well known that optical power-dependent photocurrent noise is strictly related to the statistics of impinging photons or, equivalently, to statistical properties of the detected optical fields [8]. Thus, noise in photodetection can be correctly inferred only by exploiting the amount of "optical noise" that is converted into electrical noise at the detector. In this view, the attribution of a shot-noise term $2eI_{\mathrm{ph}}$ to the photodetection of a generic optical field yielding a mean dc photocurrent I_{ph} is not always correct. For example, when detection of a coherent state is considered, the shot-noise term can be explained either by conversion at the detector of the *optical* noise $2h\nu P_s$ derived by quantum optics, or by assuming a classical noiseless sinusoidal optical wave impinging on a photocathode and attributing the noise to the quantization of the electric charge which results in Poisson statistics for the photoelectrons [16]. Among these two pictures, only the former is correct, since it is also valid for non-Poissonian photon statistics, which is really the case when amplified optical signals are considered. It is well established that both amplified signal and output ASE do not exhibit Poisson statistics [6], [17]–[19], hence the procedure followed by the standard beating theory is quite questionable.

There are also other arguments which suggest that standard beating theory is inaccurate. First, this wave-like theory uses results that are derived by a particle-like approach, thus impeding a unitary description within a well specified frame, totally wave-like or particle-like. Second, the attribution to the amplified signal of a shot noise term $2eGI_s$ is in contrast with the fact that, when a shot noise-limited signal (carrying the noise $2h\nu P_s$) is amplified by the factor G, at the output there shall be found the amplified shot noise $2h\nu G^2 P_s$. Also the terminology of beating theory is inappropriate, as the photodetection is not necessary to generate the so-called beating terms: they already exist physically in the Poynting vector whose modulus is $E \cdot E^*/2Z_0$.

Finally, it could be concluded that the standard beating approach is not a rigorous noise theory, since it fails in obtaining the correct result in a consistent way. Olsson's approach has been later reported in textbooks [1] and rearranged by other authors. In [10], the same procedure of [9] is followed with a different formalism; in [11], the so-called shot-noise terms are disregarded because negligible if compared with beating terms, thus assuming *a priori* that the exact quantum-mechanical result can be dropped.

A different semiclassical description of noise in OA's has been presented by Berglind and Gillner [14] which is based on an electrical scattering matrix equivalence for the optical gain medium, modeled in terms of negative and positive conductances representing gain/stimulated emission and optical loss/absorption, respectively. Noise generators of white spectral density proportional to $h\nu|G|$ are associated with the conductance G. This result, derived from [15], is an indirect consequence of the existence of quantum vacuum fluctuations. An expression is obtained [14, eq. (19)] accounting for the spectral density of the excess noise field spontaneously emitted by the amplifier that is a correct result differing in principle from Olsson's, as it will be shown in Section IV. However, after this result, the authors of [14] make an arbitrary transformation which allows them to recover the conventional ASE spectral density in agreement with [9], to which the vacuum fluctuation spectral density is added. In this step, the negative conductance G (gain) is formally represented as $2G + |G|$ (twice the gain plus a loss), and the existence of two uncorrelated noise sources of spectral intensity $h\nu 2|G|$ and $h\nu|G|$ is assumed, hence obtaining a total noise spectral density $h\nu(2|G| + |G|) = h\nu 3|G|$. This is an arbitrary assumption, since the total quantum noise emitted by a well-defined optical gain element (the negative conductance) is altered (multiplied by 3). When one associates noise sources to active or dissipative elements, only real elements are to be considered, and formal manipulation on their value is not allowed. As a result, also in [14] the dominant noise term is the conventional signal–ASE beating. Moreover, also in this approach input signal noise and its amplification are neglected.

The considered semiclassical analyses [9], [14] yield numerically correct results but are affected by some inconsistencies. For this reason, we intend to present a new simple and rigorous description of noise in OA's, capable of yielding the correct results in a consistent way within a semiclassical framework.

III. A New Semiclassical Wave-Theory Model

Since at optical frequencies quantum noise predominates over thermal noise, quantum-mechanical principles have to be taken into account. In [14], the noise associated with a conductance is a consequence of vacuum fluctuations [15]. Therefore, it is interesting to ask if a semiclassical theory can be developed starting directly from this very fundamental principle, as we do in the following.

The zero-mean electric field coherent (or vacuum) fluctuation is independent of the field expectation value and its variance is given by [8], [20], [21]

$$\langle \Delta E_{\mathrm{coh}}^2 \rangle = (2Z_0/A)\tfrac{1}{2}h\nu B \tag{14}$$

$$I_{\mathrm{ph}}(t) = \left(\frac{e}{h\nu}\right)\Bigg\{GP_s + 2\sqrt{4GP_sN_{\mathrm{sp}}(G-1)h\nu\delta\nu}\Big\langle\cos(2\pi\nu_s t)\cdot\sum_{k=-M,+M}\cos[2\pi(\nu_s+k\delta\nu)t+\phi_k]\Big\rangle + 2N_{\mathrm{sp}}(G-1)h\nu\delta\nu\Big\langle\sum_{k=-M,+M}\cos[2\pi(\nu_s+k\delta\nu)t+\phi_k]\sum_{l=-M,+M}\cos[2\pi(\nu_s+l\delta\nu)t+\phi_l]\Big\rangle\Bigg\} \tag{11}$$

to which corresponds the (double-sided) white power spectral density

$$S_{\Delta E_{\rm coh}}(f) = S_{\Delta E_{\rm vac}}(f) = \tfrac{1}{4}h\nu \tag{15}$$

where the factor $2Z_0/A$ has been neglected. Equation (14) comes from the minimum-uncertainty relation $\epsilon\langle\Delta E^2_{\rm coh}\rangle V = \frac{1}{2}h\nu$ for each polarization of the field, where V is the quantization volume, i.e., $V = AcT$, A is the mode (or detector) area, and $T = 1/2B$ is the observation time. Letting $c = 1/\sqrt{(\epsilon\mu)}$ and $Z_0 = \sqrt{(\mu/\epsilon)}$, (14) is obtained.

In our formulation, we try to assume the minimum number of statements, listed below, coming from quantum theory which are needed to supplement the classical description so as to obtain a consistent framework capable of treating all cases.

1) The electric field fluctuation $\Delta E_{\rm coh}$ of a coherent state has zero mean value and a variance $\langle\Delta E^2_{\rm coh}\rangle = (2Z_0/A)\frac{1}{2}h\nu B$, corresponding to a double-sided white spectral density of $\frac{1}{4}h\nu$, irrespective of the field amplitude E. Therefore, also for $E = 0$ (vacuum state or unused input port) a fluctuation $\Delta E_{\rm vac} = \Delta E_{\rm coh}$ is found and shall be taken into account.
2) The variance of the vacuum state is not directly observable by a photodetector, i.e., the variance $\langle\Delta E^2_{\rm vac}\rangle$ and higher order moments of $\Delta E_{\rm vac}$ shall be ignored when beating on a photodetector. This statement was made, in a slightly different context, by Loudon [8] and accounts for the fact that, when only the vacuum field impinges on the detector, the photocurrent is identically zero for both dc signal and noise components.
3) The fluctuation of the vacuum state shall be added to each mode and to any input port of the physical experiment at hand, even if the mode or port are unused.

We will illustrate the application of these statements with simple examples before using them later to describe the OA noise.

A. Detection of a Coherent Signal

Let us consider the model of a photodetector with quantum efficiency η receiving a coherent-state signal of signal power P_s. We assume that the total field is the sum of a deterministic signal field E_s and a zero-mean fluctuation ΔE_s. The deterministic signal is written as

$$E_s = \sqrt{(2P_s)}\cdot \exp\, i(\omega_s t + \phi_s). \tag{16}$$

The superposed fluctuation is that of the coherent state, $\Delta E_s = \Delta E_{\rm coh}$, with zero mean value $\langle\Delta E_{\rm coh}\rangle = 0$ and the variance (14). This fluctuation is represented mathematically in the form of a Gaussian stochastic variable (in agreement with [8]) having a white double-sided spectral density given by (15).

The photodetector with nonunitary quantum efficiency $(\eta < 1)$ is a port for the vacuum state, and for clarification we shall model it as an ideal $(\eta' = 1)$ photodetector preceded by a beamsplitter with power transmission η [22]. The beamsplitter will therefore transmit a fraction $\sqrt{\eta}$ of the signal field and reflect a fraction $i\sqrt{(1-\eta)}$ (phase-shifted of 90°) of the vacuum field $\Delta E_{\rm vac}$ entering this unused port, as in Fig. 1.

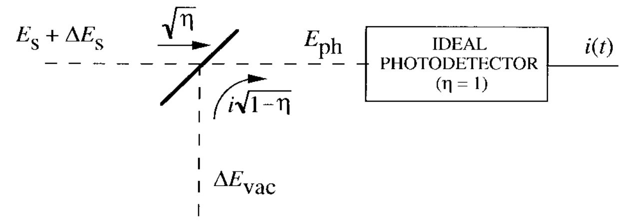

Fig. 1. Model for the photodetection of a coherent signal with quantum efficiency η through a beamsplitter of field transmission $\sqrt{\eta}$.

Here, $\Delta E_{\rm vac}$ has the same statistical properties of ΔE_s, but these two variables are statistically uncorrelated.

Accordingly to statement 2) made above, the photodetected current $I_{\rm ph}(t)$ is obtained by subtraction of a term $\langle\Delta E^2_{\rm vac}\rangle$ from the squared modulus of the total electric field impinging on the photodetector, given by $E_{\rm ph} = \sqrt{\eta}\cdot(E_s + \Delta E_{\rm coh}) + i\sqrt{1-\eta}\cdot\Delta E_{\rm vac}$. Thus, we have for the dc term

$$\begin{aligned}\langle I_{\rm ph}(t)\rangle &= (e/h\nu)[\langle|E_{\rm ph}(t)|^2\rangle - \langle|\Delta E_{\rm vac}(t)|^2\rangle] \\ &= (e/h\nu)[\eta P_s + \eta\langle|\Delta E_{\rm coh}(t)|^2\rangle \\ &\qquad + (1-\eta)\langle|\Delta E_{\rm vac}(t)|^2\rangle \\ &\qquad - \langle|\Delta E_{\rm vac}(t)|^2\rangle] \\ &= (e/h\nu)\eta P_s = \eta I_s. \end{aligned}\tag{17}$$

The spectral density of the zero-mean photocurrent fluctuation $I_{{\rm ph},n}(t) = I_{\rm ph}(t) - \langle I_{\rm ph}(t)\rangle$ can be obtained by the Fourier transform of the photocurrent autocovariance function $\boldsymbol{C}_{I_{\rm ph}}(\tau) = \langle I_{{\rm ph},n}(t)\cdot I_{{\rm ph},n}(t+\tau)\rangle$ [12] which is given by

$$\begin{aligned}\boldsymbol{C}_{I_{\rm ph}}(\tau) = \left(\frac{e}{h\nu}\right)^2 \{&\eta^2\boldsymbol{C}_{|\Delta E{\rm coh}|^2}(\tau) + (1-\eta)^2\boldsymbol{C}_{|\Delta E{\rm vac}|^2}(\tau) \\ &+ 4\eta^2\boldsymbol{C}_{Es\cdot\Delta E{\rm coh}}(\tau) \\ &+ 4\eta(1-\eta)\boldsymbol{C}_{Es\cdot\Delta E{\rm vac}}(\tau) \\ &+ 4\eta(1-\eta)\boldsymbol{C}_{\Delta E{\rm coh}\cdot\Delta E{\rm vac}}(\tau) \\ &- \boldsymbol{C}_{|\Delta E{\rm vac}|^2}(\tau)\}\end{aligned}\tag{18}$$

where it is assumed that $\boldsymbol{C}_{|E|^2}(\tau)$ is the autocovariance of the function $|E(t)|^2$ and $\boldsymbol{C}_{E1(t)\cdot E2(t)}(\tau)$ that of the function ${\rm Re}[E1(t)\cdot E2(t)]$. Equation (18) can be simplified using the properties of autocovariance functions and keeping in mind that the fields $\Delta E_{\rm vac}(t)$ and $\Delta E_{\rm coh}(t)$ are uncorrelated, i.e., the term $\boldsymbol{C}_{|\Delta E{\rm vac}|^2}(\tau)$ yields a noise spectral density twice the one given by $\boldsymbol{C}_{\Delta E{\rm coh}\cdot\Delta E{\rm vac}}(\tau)$. Thus, the first, second, fifth, and sixth terms of (18) cancel out, while the third and fourth terms yield a white photocurrent noise with single-sided spectral density given by

$$S_{I{\rm ph},n}(f) = 2e\eta I_s \tag{19}$$

which is the correct shot noise for the considered case. Note that, from the semiclassical point of view, by reversing the arguments, the $(1/4)h\nu$ double-sided spectral density of the coherent state fluctuation (15) can be invoked as necessary for the shot noise to be given by (19). Also, the amplitude fluctuation of the detected current, which is known to be Gaussian in the time domain (i.e., for the random variable $I_{{\rm ph},n}(t)$) and a negative exponential in the frequency domain (i.e., for the random instantaneous power) correctly comes out, since the coherent state fluctuation $\Delta E_{\rm coh}(t)$ is assumed to be Gaussian-distributed [23].

B. Detection of a Laser Signal with Excess-Noise

A real laser source does not emit a pure coherent state and its photodetection noise is larger than pure shot noise. Also, the correct modeling of a real source is of importance for the evaluation of the performances of a communication fiber link including OA's. We can easily model such a source as a deterministic field E_s given by (16) to which a field fluctuation $\Delta E_{\rm exc}(t)$ is superposed. The double-sided spectral density of $\Delta E_{\rm exc}(t)$ is given by

$$S_{\Delta E_{\rm vac}}(f) = \tfrac{1}{4}h\nu\{1 + k\cdot {\rm rect}[-B_x/2, +B_x/2] \otimes [\delta(\nu_s) + \delta(-\nu_s)]\} \tag{20}$$

where $k \geq 0$ and $\otimes$ denotes convolution. The total field spectral density of the nonideal laser source is shown in Fig. 2. The two rectangular-shaped noise densities in excess of the vacuum fluctuation account for the ASE power which is always emitted by a real source. A rectangular-shaped optical gain is schematically assumed for the laser medium with a bandwidth equal to B_x. Photodetection of the considered signal yields the following dc current:

$$\begin{aligned}\langle I_{\rm ph}(t)\rangle &= (e/h\nu)[P_s + k\cdot B_x \cdot h\nu/2]\\ &= (e/h\nu)[P_s + P_{{\rm ASE},l}]\end{aligned} \tag{21}$$

where the CW ASE power emitted by the laser $P_{{\rm ASE},l}$ is negligible for usual cases. The single-sided spectral density of the noise current is now given by

$$\begin{aligned}S_{I{\rm ph},n}(f) = (e/h\nu)^2\{&2h\nu(P_s + P_{{\rm ASE},l}) + 2h\nu k P_s\\ &\cdot {\rm rect}[0, B_x/2] + 2P^2_{{\rm ASE},l}/B_x\\ &\cdot (1 - f/B_x)\cdot {\rm rect}[0, B_x]\}\end{aligned} \tag{22}$$

and we have

$$\begin{aligned}S_{I{\rm ph},n}(0) &= (e/h\nu)^2\{2h\nu[(k+1)P_s + (k/2+1)P_{{\rm ASE},l})]\}\\ &\approx 2e(k+1)I_s\end{aligned} \tag{23}$$

where the approximation holds for most cases. It can be easily seen from (23) that the quantity $(k+1)$ represents the Fano factor of the considered real source

$$F_{\rm source} = k + 1 \tag{24}$$

or, equivalently, it is the ratio between the source RIN and the RIN of a pure coherent source. The above description correctly accounts for the evolution of the Fano factor of the light transmitted through an attenuator [21], which can be represented by a beamsplitter as in Section III-A.

C. Model of the Optical Amplifier

The key point for the new description is that the additional noise responsible for the ASE enters the optical amplifier from an unused port. This statement is consistent with quantum theory prohibiting an ideal two-port amplifier (i.e., with just input and output). Rather, if the field-amplification coefficient from input to output is $\sqrt{G}$, quantum theory requires that there shall be another port from which noise enters the optical amplifier, with a field-transfer coefficient $i\sqrt{(G-1)}$. Indeed, if A is the signal field at the physical input port field and A' is

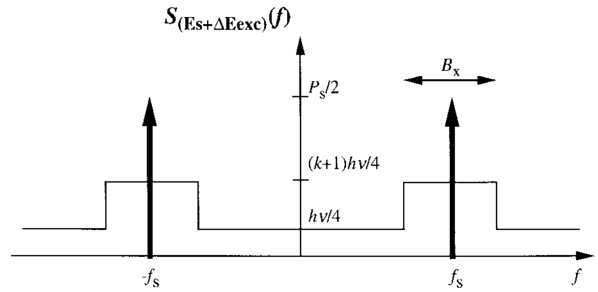

Fig. 2. Spectral density of the total field emitted by a laser source with Fano factor $F_{\rm source} = k + 1$.

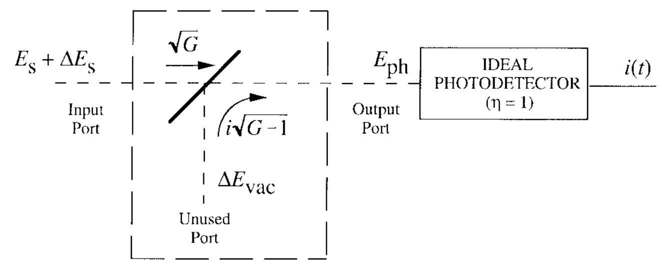

Fig. 3. Model of an optical amplifier of gain G and complete medium inversion ($N_{\rm sp} = 1$) represented as an amplifying beamsplitter of field transmission $\sqrt{G}$. At the idler port, the vacuum field fluctuation is applied.

the field at the idler input, the commutation relations require [22], [24] that the transfer matrix be of the form

$$\begin{bmatrix} B \\ B' \end{bmatrix} = \begin{bmatrix} \sqrt{G} & i\sqrt{G-1} \\ i\sqrt{G-1} & \sqrt{G}\end{bmatrix}\begin{bmatrix} A \\ A'\end{bmatrix}. \tag{25}$$

This matrix is identical to that of a beamsplitter with a fictitious $G > 1$ factor of power transmittance. From the relationship point of view, the transfer matrix allows us to describe the optical amplifier as a four-port device, as shown in Fig. 3. We can again represent a coherent input signal E_s and its fluctuation $\Delta E_s(t) = \Delta E_{\rm coh}(t)$, as in Section III-A, and assume that the vacuum state enters the idler port with zero mean value and a white spectral density given by (15). The total electric field on the photodetector (assumed to have unitary quantum efficiency) is now given by $E_{\rm ph} = \sqrt{G(f)}\cdot(E_s + \Delta E_{\rm coh}) + i\sqrt{G(f)-1}\cdot\Delta E_{\rm vac}$. The spectral dependence of the optical gain is $G(f) = G$ for $f_s - B_o/2 < f < f_s + B_o/2$, and $G(f) = 1$ for $f < f_s - B_o/2$ and $f > f_s + B_o/2$ where the active medium is transparent. For the moment, let us consider an OA with complete medium inversion, i.e., $N_{\rm sp} = 1$.

The photocurrent is obtained according to statement 2), and the dc value is given by (26), shown at the bottom of the page, and coincides with (3). The spectral density of the zero-mean photocurrent fluctuation $I_{{\rm ph},n}(t) = I_{\rm ph}(t) - \langle I_{\rm ph}(t)\rangle$ is obtained by the Fourier transform of the photocurrent autocovariance function $\boldsymbol{C}_{I_{\rm ph}}(\tau)$ which is now given by

$$\begin{aligned}\boldsymbol{C}_{I{\rm ph}}(\tau) = \left(\frac{e}{h\nu}\right)^2\{&\boldsymbol{C}_{G(f)\cdot|\Delta E{\rm coh}|^2}(\tau)\\ &+ \boldsymbol{C}_{[G(f)-1]\cdot|\Delta E{\rm vac}|^2}(\tau)\\ &+ 4\boldsymbol{C}_{G(f)\cdot Es\cdot\Delta E{\rm coh}}(\tau)\\ &+ 4\boldsymbol{C}_{\sqrt{G(f)[G(f)-1]}\cdot Es\cdot\Delta E{\rm vac}}(\tau)\\ &+ 4\boldsymbol{C}_{\sqrt{G(f)[G(f)-1]}\cdot\Delta E{\rm coh}\cdot\Delta E{\rm vac}}(\tau)\\ &- \boldsymbol{C}_{|\Delta E{\rm vac}|^2}(\tau)\}.\end{aligned} \tag{27}$$

The Fourier transform of the first, second, fifth and sixth terms of (27) gives the noise associated to ASE. The ASE is not postulated in our model but rather comes out as the amplification of coherent signal fluctuations and vacuum state fluctuations. The ASE noise has a single-sided spectral density (28), shown at the bottom of the page. The third term of (27) gives the noise due to the amplification of the input signal coherent state fluctuation, which has a single-sided spectral density

$$S_{I\mathrm{ph,amp}}(f) = \begin{cases} (e/h\nu)^2 \cdot 2h\nu G^2 P_s = 2eG^2 I_s, \\ \quad 0 < f < B_o/2 \\ (e/h\nu)^2 \cdot 2h\nu G P_s = 2eGI_s, \\ \quad f > B_o/2 \end{cases}. \tag{29}$$

The fourth term of (27) yields the noise due to the amplification of the vacuum state fluctuations entering the amplifier idler port. This constitutes amplifier excess noise with a spectral density

$$S_{I\mathrm{ph,exc}}(f) = \begin{cases} (e/h\nu)^2 \cdot 2h\nu G(G-1)P_s = 2eG(G-1)I_s, \\ \qquad 0 < f < B_o/2 \\ 0, \quad f > B_o/2 \end{cases}. \tag{30}$$

The spectral densities (28)–(30) are reported in Fig. 4.

The total photocurrent noise spectral density is $S_{I\mathrm{ph}}(f) = S_{I\mathrm{ph,ASE}}(f) + S_{I\mathrm{ph,amp}}(f) + S_{I\mathrm{ph,exc}}(f)$ and agrees with the correct expression (4) and with the conventional result [9]. The approach presented here suggests a clear interpretation for the OA output noise terms. Neglecting the term (28), the dominant noise is represented by two terms: 1) the amplification of input signal coherent fluctuation $2eG^2I_s$ (equation (29), amplified shot noise) and 2) the excess-noise $2eG(G-1)I_s$ introduced by the amplifier. For $G \gg 1$, these two terms have the same magnitude and the 3-dB minimum noise figure is obtained. The high-frequency white noise terms $2eI_{\mathrm{ASE}}, 2eGI_s$ cannot be physically regarded as ASE and amplified signal shot noises, since they extend well beyond the limit for which electronic formalism is valid. Further statistical properties of output noise can be inferred from the structure of the terms of (27), keeping in mind that the fluctuations $\Delta E_{\mathrm{coh}}, \Delta E_s$ and ΔE_{vac} are Gaussian-distributed. The probability density function for each term of $I_{\mathrm{ph},n}(t)$ can be obtained, compounding the results with standard statistical rules.

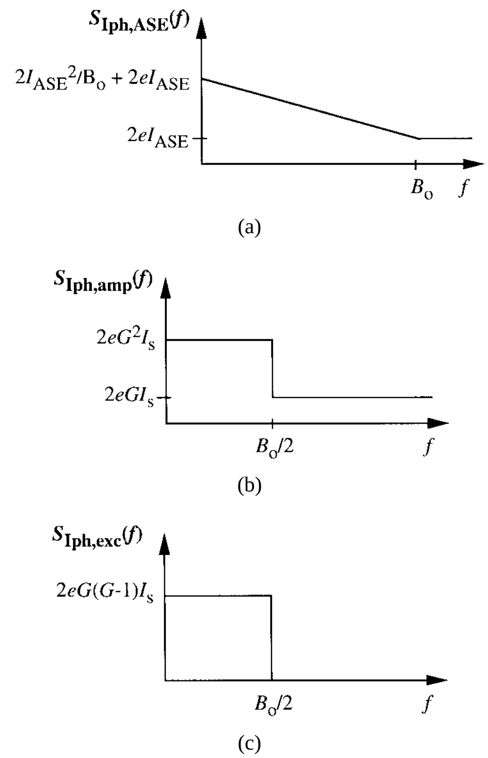

Fig. 4. Spectral density of OA output photocurrent noise terms (28)–(30). (a) ASE noise, (b) amplified input-signal noise, and (c) amplifier excess noise.

D. Extensions of the Results

To avoid excessive burdening of the notation, in the previous section we have assumed ideal photodetector quantum efficiency, a coherent input signal, and unitary ASE factor N_{sp}. Now we can generalize the above results dropping these assumptions.

First, let us note that a subunitary quantum efficiency of the photodetector has nothing to do with amplifier noise, and it is therefore reasonable to model it by a beamsplitter (as in Section III-A) cascaded to the OA.

Second, the excess noise carried by the input laser signal can be modeled as in Section III-B, thus obtaining the same result given by the quantum theory when an input signal Fano factor $F_i > 1$ is assumed.

Third, to take into account incomplete inversion, i.e., $N_{\mathrm{sp}} > 1$, a sort of ASE amplification shall be introduced.

$$\begin{aligned} \langle I_{\mathrm{ph}}(t)\rangle &= (e/h\nu)[\langle|E_{\mathrm{ph}}(t)|^2\rangle - \langle|\Delta E_{\mathrm{vac}}(t)|^2\rangle] \\ &= (e/h\nu)\{\langle|\sqrt{G(f)}\cdot E_s(t)|^2\rangle + \langle|\sqrt{G(f)}\cdot \Delta E_{\mathrm{coh}}(t)|^2\rangle + \langle|\sqrt{G(f)-1}\Delta E_{\mathrm{vac}}(t)|^2\rangle - \langle|\Delta E_{\mathrm{vac}}(t)|^2\rangle\} \\ &= (e/h\nu)[GP_s + \langle|\Delta E_{\mathrm{coh}}(t)|^2\rangle + \tfrac{1}{2}h\nu(G-1)B_o + \tfrac{1}{2}h\nu(G-1)B_o - \langle|\Delta E_{\mathrm{vac}}(t)|^2\rangle] \\ &= (e/h\nu)[GP_s + h\nu(G-1)B_o] \\ &= GI_s + I_{\mathrm{ASE}} \end{aligned} \tag{26}$$

$$\begin{aligned} S_{I\mathrm{ph,ASE}}(f) &= (e/h\nu)^2 \cdot \{2(h\nu)^2(G-1)^2B_o \cdot (1 - f/B_o)\cdot \mathrm{rect}[0, B_o] + 2(h\nu)^2(G-1)B_o\} \\ &= 2I_{\mathrm{ASE}}^2/B_o \cdot (1 - f/B_o)\cdot \mathrm{rect}[0, B_o] + 2eI_{\mathrm{ASE}} \end{aligned} \tag{28}$$

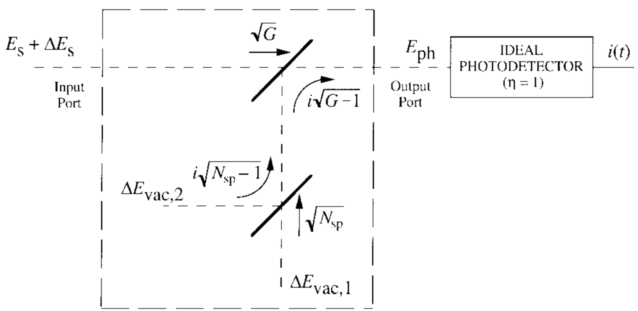

Fig. 5. Model of an optical amplifier of gain G and incomplete medium inversion ($N_{sp} \geq 1$).

Since the ASE comes from the unused port, an amplifying beamsplitter with a field transmission $\sqrt{N_{sp}}$ and a field reflection $i\sqrt{(N_{sp}-1)}$ shall be added on the idler port, as shown in Fig. 5. To this beamsplitter, two input uncorrelated fluctuations ΔE_{vac1} and ΔE_{vac2} are applied. With this new scheme, it is an exercise to repeat the calculations as in Section III-C, and the results for the dc photocurrent is found coincident to (26), with an ASE CW power now given by $P_{ASE} = h\nu N_{sp}(G-1)B_o$. The ASE photocurrent noise term has now the form

$$\begin{aligned} S_{I\text{ph,ASE}}(f) &= (e/h\nu)^2 \\ &\quad \cdot \{2(h\nu)^2 N_{sp}^2 (G-1)^2 B_o \\ &\quad \cdot (1 - f/B_o) \cdot \text{rect}[0, B_o] \\ &\quad + 2(h\nu)^2 N_{sp}(G-1)B_o\} \\ &= 2I_{ASE}^2/B_o \cdot (1 - f/B_o) \\ &\quad \cdot \text{rect}[0, B_o] + 2eI_{ASE}. \end{aligned} \tag{31}$$

The noise due to the amplification of the input signal fluctuation (29) is unchanged, while the amplifier excess-noise term is now given by

$$S_{I\text{ph,exc}}(f) = \begin{cases} (e/h\nu)^2 \cdot 2h\nu(2N_{sp}-1)G(G-1)Ps \\ \quad = 2e(2N_{sp}-1)G(G-1)I_s, \\ \qquad 0 < f < B_o/2 \\ 0, \qquad f > B_o/2 \end{cases} \tag{32}$$

IV. Discussion

Our new wave-like description allows a clearer interpretation of signal-dependent OA output noise as due to amplified input signal noise and to the unavoidable excess noise introduced by the amplifier. This picture very closely resembles that of the photon probabilistic approach [6], [17]–[19] in which, apart from ASE power fluctuations, the output noise consists of amplification of the shot noise associated with the input signal and of excess noise due to amplifier gain randomness.

A further comparison between our results and those obtained in [14] is worthwhile. The spectral density of the noise field spontaneously emitted by the OA is given in [14, eq. (19)]. This expression is a correct result and corresponds to the output optical spectral density obtained in our approach by amplification of the vacuum fluctuation entering the amplifier idler port. In our model, this noise field contributes, by means of interference with the amplified signal and with the amplified noise carried by the signal itself, to the total output CW and fluctuation power. This gives rise to the correct CW and noise terms. In [14], input field fluctuations (that are at least equal to vacuum fluctuations) are neglected and so is their amplification. Hence, at OA output, the total optical spectral density lacks a noise term, which is why the analysis is not fully consistent. It is now clear the importance of taking explicitly into account input signal fluctuations (or vacuum fluctuations if no signal is applied to OA input) and their amplification in order to obtain a consistent description.

When a transmission line with cascaded OA's featuring rectangular-shaped gain is considered, conventional semiclassical approaches [9], [14] give correct results, despite the inconsistencies analyzed above. However, the validity of these approaches is limited to the case of pure coherent input signal. This is a limitation that can be easily overcome by our approach. Our method also allows the characterization of a cascade of OA's with arbitrary spectral gain shape, yielding the exact ASE noise contribution which strongly depends on spectral gain shape.

When there is no need to distinguish between various noise terms, the calculation (26)–(27) can be carried out directly on the total field E_{ph}, and its whole spectral density can be considered.

V. Conclusion

We have formulated a new semiclassical wave theory of the noise in optical amplifiers which is a simple yet rigorous translation of a few quantum statements. We have shown that the OA can be modeled as a sort of amplifying beamsplitter and that the ASE comes from the amplification of vacuum fluctuations. Examples of application point out the general validity of the description.

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References

[1] E. Desurvire, *Erbium-Doped Fiber Amplifiers—Principle and Applications.* New York: Wiley, 1994.

[2] H. A. Haus and J. A. Mullen, "Quantum noise in linear amplifiers," *Phys. Rev.*, vol. 128, no. 5, pp. 2407–2413, 1962.

[3] C. M. Caves, "Quantum limits on noise in linear amplifiers," *Phys. Rev. D*, vol. 26, no. 8, pp. 1817–1839, 1982.

[4] Y. Yamamoto and T. Mukai, "Fundamentals of optical amplifiers," *Opt. Quantum Electron.*, vol. 21, pp. s1–s14, 1989.

[5] O. Hirota, Ed., *Squeezed Light.* Amsterdam, The Netherlands: Elsevier, 1992.

[6] K. Shimoda, H. Takahasi, and C. H. Townes, "Fluctuations in amplification of quanta with application to master amplifier," *J. Phys. Soc. Japan*, vol. 12, no. 6, pp. 686–700, 1957.

[7] Y. Yamamoto, "Noise and error rate performance of semiconductor laser amplifiers in PCM-IM optical transmission systems," *IEEE J. Quantum Electron.*, vol. QE-16, no. 10, pp. 1073–1080, 1980.

[8] R. Loudon, *The Quantum Theory of Light.* Oxford: Clarendon Press, 1983.

[9] N. A. Olsson, "Lightwave systems with optical amplifiers," *J. Lightwave Technol.*, vol. 7, pp. 1071–1082, 1989.

[10] R. Ramaswami and P. A. Humblet, "Amplifier induced crosstalk in multichannel optical networks," *J. Lightwave Technol.*, vol. 8, pp. 1882–1896, 1990.

[11] J. T. Kringlebotn, K. Bltekjaer, and C. N. Pannell, "Field statistics modeling of beat noise in an optical amplifier," in *Proc. Inst. Elect. Eng.-Optoelectron.*, vol. 141, no. 3, pp. 185–190, 1994.

[12] W. B. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise*. New York: McGraw-Hill, 1958.

[13] B. M. Oliver, "Thermal and quantum noise," *Proc. IEEE*, vol. 53, pp. 436–454, 1965.

[14] E. Berglind and L. Gillner, "Optical quantum noise treated with classical electrical network theory," *IEEE J. Quantum Electron.*, vol. 30, pp. 846–853, 1994.

[15] O. Nilsson, Y. Yamamoto, and S. Machida, "Internal and external field fluctuations of a laser oscillator—Part II," *IEEE J. Quantum Electron.*, vol. QE-22, pp. 2043–2051, 1986.

[16] A. Yariv, *Optical Electronics*, 3rd ed. New York: Holt-Saunders International Editions, 1985, chs. 10–11.

[17] T. Li and M. C. Teich, "Photon point process for traveling-wave laser amplifier," *IEEE J. Quantum Electron.*, vol. 29, no. 9, pp. 2568–2578, 1993.

[18] G. L. Cariolaro, P. Franco, M. Midrio, and G. L. Pierobon, "Complete statistical characterization of signal and noise in optically amplified fiber channels," *IEEE J. Quantum Electron.*, vol. 31, pp. 1114–1122, 1995.

[19] P. Diament and M. C. Teich, "Evolution of the statistical properties of photons passed through a traveling-wave laser amplifier," *IEEE J. Quantum Electron.*, vol. 28, pp. 1325–1334, 1992.

[20] A. Yariv, *Quantum Electronics*, 3rd ed. New York: Wiley, 1989, ch. 5.8.

[21] V. Annovazzi-Lodi, S. Donati, and S. Merlo, "Squeezed states in direct and coherent detection," *Opt. Quantum Electron.*, vol. 24, pp. 285–301, 1992.

[22] C. W. Gardiner, *Quantum Noise.* Berlin, Germany: Springer-Verlag, 1991.

[23] A. Papoulis, Probability, *Random Variables and Stochastic Processes*. New York: McGraw-Hill, 1955, ch. 13.1.

[24] G. M. D'Ariano, "Number-phase squeezed states and photon fractioning," *Int. J. Mod. Phys. B*, vol. 6, pp. 1291–1354, 1992.

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