Mode-Resolved Measurements of the Linewidth Enhancement Factor of a Fabry–Pérot Laser

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***Abstract*—The linewidth enhancement factor of a multiple longitudinal-mode laser is measured using both the traditional Hakki–Paoli method and two proposed modifications to techniques that are generally used for single longitudinal-mode lasers. Parameters are measured separately for the different modes and compared.**

***Index Terms*—Laser measurements, semiconductor lasers.**

I. INTRODUCTION

LINEWIDTH enhancement factor [1], widely referred to as α-factor, is a parameter of major relevance when characterizing semiconductor lasers as it affects fundamental aspects of its behavior, such as linewidth, modulation-induced chirp, mode stability, and other nonlinear phenomena.

Estimation of the α-factor from measurements of the gain (Hakki–Paoli (HP) method [2]) has been traditionally used to characterize multiple longitudinal-mode lasers and gain materials in general. Due to their simplicity of implementation requiring only optical spectrum measurements, methods based on measuring the gain have been very widespread and several implementations can be found in literature [3], [4]. These measurements lead to the so-called material α-factor, which is dependent on the emitted wavelength.

When it comes to analyze the dynamics of single longitudinal-mode lasers, the influence of the α-factor on the laser dynamics can be used, and many methods have been developed. Interaction between chirp and chromatic dispersion [5], linewidth [6], phase-to-amplitude ratio [7], or the behavior under feedback [8] can be used to measure the "device α-factor" that gives a single value for the lasing mode.

Although material α-factor should, in principle, be able to predict the device α-factor, large discrepancies are usually found in practice. In a previous work conducted within European COST Action 288, a round-robin activity was developed on the measurement of the α-factor on distributed feedback lasers, with results showing a consistent underestimation of the α-factor when using gain measurements in comparison with other methods [9].

In this letter, we take a different approach to this problem and propose two alternative methods for the measurement of the α-factor in multiple longitudinal-mode lasers by implementing modifications to methods that were originally developed for single longitudinal-mode lasers. These measurements are performed separately over each lasing mode, thus obtaining an α-factor value for each longitudinal mode that will be compared with the material α-factor at that wavelength.

II. THEORETICAL BASIS

The linewidth enhancement factor was originally introduced by Henry [1] to explain the linewidth broadening occurring in semiconductor lasers compared to the conventional Schawlow–Townes formula, valid for gas lasers [10]. According to Henry's explanation, the coupling of phase and intensity produces a broadening of the linewidth by a factor $(1+\alpha^2)$, with α being the linewidth enhancement factor.

The α-factor is thus defined as the ratio of the partial derivatives of the real and complex parts of the complex susceptibility $\chi = \chi_r + i\chi_i$ with respect to carrier density N

$$\alpha = -\frac{\partial \chi_r / \partial N}{\partial \chi_i / \partial N} = -\frac{4\pi}{\lambda}\frac{dn}{dg} \tag{1}$$

where dn and dg are the small index and optical gain variations that occur for a carrier density variation dN.

For this definition to be valid in order to assess frequency chirp effects, the time scale of the changes must be considered, as several physical causes acting on very different time scales contribute to the variation of the refractive index, such as band filling, free carrier contribution, and thermal effects [11]. In particular, thermal effects may alter the output of some methods if they are not properly compensated for [4].

Measurement methods developed for single longitudinal-mode lasers assume a high degree of stability of the laser, which may not be the case for each of the modes of a Fabry–Pérot, as mode partitioning is present. However, for the methods considered here, these dynamics are integrated over the measurement averaging time, and are slow enough not to interact with the high-frequency modulation [12].

III. EXPERIMENT

The linewidth enhancement factor has been measured for seven adjacent modes of a Fabry–Pérot laser using three different methods: HP gain measurement, fiber transfer function (FTF) measurement, and linewidth power ratio (LPR). Seven additional modes have been characterized with the first two methods.

The measured laser's central wavelength is located at 1545 nm, with modes separated roughly by 1 nm. Threshold

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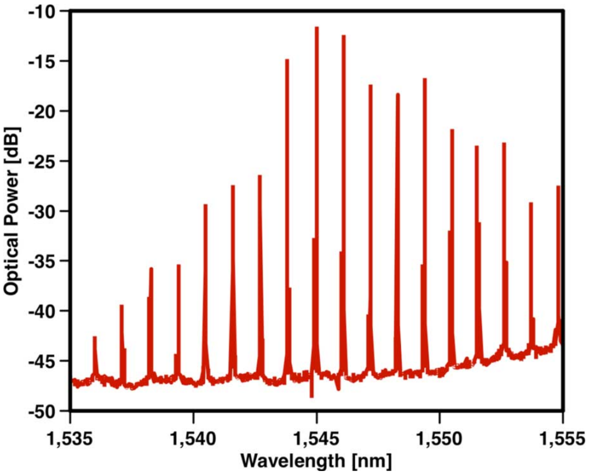

Fig. 1. Measured optical spectrum of the laser biased at 15 mA.

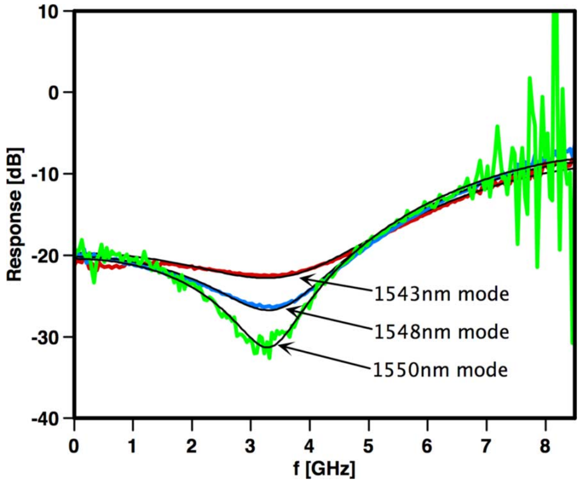

Fig. 2. Sample measurement of the FTF for three different modes obtained with 50 km of single-mode fiber (16 averages) and fittings to (4).

current ranges from 8 to 13 mA depending on device temperature. A typical optical spectrum of the laser is shown in Fig. 1.

A. Hakki–Paoli

The HP method is based on the measurement of the gain from the relation between peaks and valleys of the amplified spontaneous emission of the laser operated below threshold and its variations as the modulation intensity changes. A set of spectra is thus measured with an optical spectrum analyzer (OSA) while sweeping the current from threshold down to the lowest current value that generates a measurable spectrum.

In order to reduce the effect of the wavelength resolution of the OSA, the maximum/minimum ratio is replaced with the relation between the integral of the optical power of the mode and its minimum value [3]. This new ratio (ρ') is then scaled by the free spectral range of the resonator $(\rho = \rho' 2L/c)$, where L is the cavity length, thus allowing to compute the mth-mode single-pass gain (G_m) as

$$RG = \frac{\rho - 1}{\rho + 1} \tag{2}$$

where R is the facet reflectivity. It can be noted that G_m reaches saturation as bias intensity approaches threshold. The modal gain g_m is then computed by applying a logarithmic relation to *RG*

$$\frac{1}{L}\ln\left(\frac{RG}{R}\right) = \Gamma g_m - \alpha_{\text{loss}} \tag{3}$$

where Γ is the confinement factor and α_{loss} accounts for the cavity losses. To avoid border effects arising when working with a staggered function in the following steps, a smoothing is performed prior to any further mathematical transformation.

Then, we use the fringe separation to calculate refractive index, correcting for thermal effects using the procedure described by Rodriguez *et al.* [4]. This procedure is essential to avoid thermal effects to alter the estimate of the α-factor.

B. Fiber Transfer Function

The interaction of chirped light with fiber dispersion can also be used to measure the α-factor from the measured FTF. With the aid of a network analyzer, the transfer function of a dispersive medium, such as an optical fiber, can be also measured. The interaction of dispersion and phase modulation due to the laser chirp when the laser bias current is modulated will configure a frequency response expressed by [5]

$$H(f) = \cos\theta - \alpha\left(1 - j\frac{f_c}{f}\right)\sin\theta \tag{4}$$

with $\theta = f^2\pi\lambda^2 Dl/c$, where f_c is the chirp frequency, accounting for adiabatic chirp, D is the dispersion coefficient, and l is the fiber length.

A calibration of the system is made directly detecting the emitted light; the dispersive medium is inserted afterward to get the transfer function. As demonstrated in [10], the conventional FTF method cannot be directly applied to a multimode laser, as the response of all the lasing modes are superoposed. To avoid this inconvenience, we used a tunable optical filter with 0.4 nm bandwidth to select a single mode of the laser. An OSA is used to verify the adequate filtering.

Measurements and fittings of the FTF for several laser modes are depicted in Fig. 2. Fittings to (4) seem precise up to 7 GHz, even though the modulation bandwidth of the laser is rather low.

C. Linewidth Power Ratio

This last method is based on the measurement of the evolution of the relation between the power and the linewidth of the laser modes as the intensity moves away from the threshold region [6]. Below threshold, the linewidth of semiconductor lasers has a slope $\Delta v_<$ when measured against inverse output power defined by the Schalow–Townes relation [10], whereas over threshold, its slope $\Delta v_>$ asymptotically fits to the Henry expression [1]. From the ratio of these two slopes, the α-factor is obtained using [6]

$$\alpha = \sqrt{2\frac{\Delta v_>}{\Delta v_<} - 1}. \tag{5}$$

In order to perform the measurements, the lower power modes are amplified by an erbium-doped fiber amplifier (EDFA). Power and linewidth are measured with an Aragon Photonics BOSA-C.

Instead of performing a full intensity sweep to obtain curves like the one shown in Fig. 3, which contain a high number of

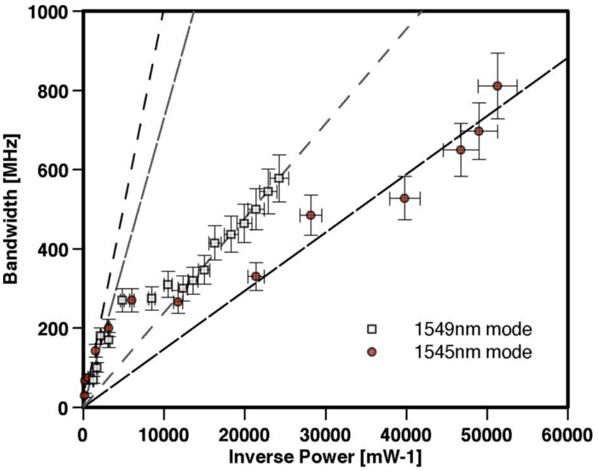

Fig. 3. Full-width at half-maximum (FWHM) linewidth versus inverse power around the threshold region for two different modes. Long dashed lines stand for the Henry asymptotic behavior [1] and short dashed lines for the Schawlow–Townes asymptotic behavior [10]. Error bars represent a 0.2 dB power uncertainity and the 95% confidence interval for the linewidth fittings.

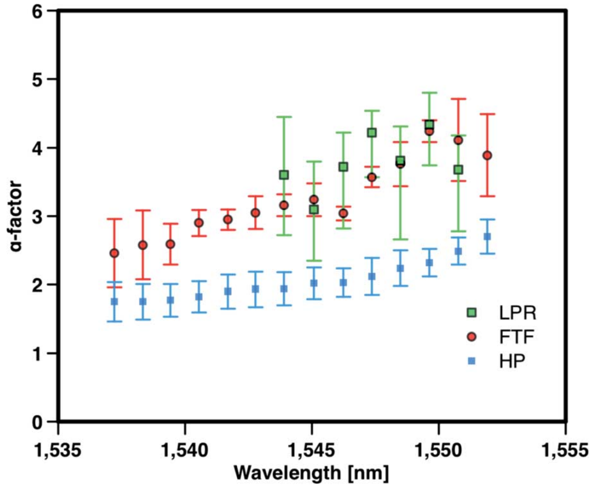

Fig. 4. Measured α-factor values with the three methods for each of the laser modes, represented with their 95% confidence intervals.

points in the middle region that do not affect the calculation of the slopes, two groups of measurements are performed, located in the regions where the asymptotical behavior has already been reached. Two clouds of points are thus obtained. The slopes are then computed considering all the points of each region.

IV. RESULTS

The results obtained with the three different methods are reported in Fig. 4. The amplitude of the error bars for the different methods are comparable with those from the LPR method being slightly larger due to the impact of the temperature drifts on the estimation of the linewidth. For the FTF method, error bars increase as the power of the mode decreases, due to the smaller signal-to-noise ratio available at the network analyzer, whereas for the HP method remain mostly constant due to gain curve smoothing. The HP method error bars include the uncertainty due to estimated laser parameters.

From the analysis of Fig. 4, two comments are in order.

1) The LPR and FTF methods give similar results, although with some fluctuation for some specific modes.
2) The HP method gives systematically smaller α values with respect to the other two methods, with differences that amount to 40% at shorter wavelengths and to 60% at longer wavelengths.

From the comparative measurements, it appears that the subthreshold HP technique (that measures the “material” alpha) underestimates the alpha factor for the lasing device. Instead, the LPR and FTF methods involve measurements above threshold, and their results seem to be better representative for the alpha value in operating conditions.

This research suggests that methods developed for single-mode lasers can be effectively applied to multiple longitudinal-mode lasers with minor modifications on the setups. It also appears that the use of these methods shall be recommended instead of the classical HP technique.

REFERENCES

[1] C. Henry, “Theory of the linewidth of semiconductor lasers,” *IEEE J. Quantum Electron.*, vol. QE-18, no. 2, pp. 259–264, Feb. 1982.

[2] B. W. Hakki and T. L. Paoli, “Gain spectra in GaAs double-heterostructure injection lasers,” *J. Appl. Phys.*, vol. 46, no. 3, pp. 1299–1306, Mar. 1975.

[3] D. T. Cassidy, “Technique for measurement of the gain spectra of semiconductor diode laser,” *J. Appl. Phys.*, vol. 56, no. 11, pp. 3096–3099, Dec. 1984.

[4] D. Rodriguez, L. Borruel, I. Esquivias, H. Wenzel, B. Sumpf, and G. Erbert, “CW technique for measurement of linewidth enhancement factor: Application to 735-nm tensile-strained GaAsP quantum-well lasers,” *IEEE Photon. Technol. Lett.*, vol. 16, no. 6, pp. 1432–1434, Jun. 2004.

[5] R. C. Srinivasan and J. C. Cartledge, “On using fiber transfer functions to characterize laser chirp and fiber dispersion,” *IEEE Photon. Technol. Lett.*, vol. 7, no. 11, pp. 1327–1329, Nov. 1995.

[6] A. Villafranca, J. A. Lázaro, I. Salinas, and I. Garcés, “Measurement of the linewidth enhancement factor in DFB lasers using a high-resolution optical spectrum analyzer,” *IEEE Photon. Technol. Lett.*, vol. 17, no. 11, pp. 2268–2270, Nov. 2005.

[7] C. Harder, K. Vahala, and A. Yariv, “Measurement of the linewidth enhancement factor α of semiconductor lasers,” *Appl. Phys. Lett.*, vol. 42, no. 4, pp. 328–330, Feb. 1983.

[8] Y. Yu, G. Giuliani, and S. Donati, “Measurement of the linewidth enhancement factor of semiconductor lasers based on the optical feedback self-mixing effect,” *IEEE Photon. Technol. Lett.*, vol. 16, no. 4, pp. 990–992, Apr. 2004.

[9] A. Villafranca, J. Lasobras, I. Garcés, and G. Giuliani *et al.*, “Linewidth enhancement factor of semiconductor lasers: Results from round-robin measurements in COST288,” presented at the Conf. Lasers Electro-Opt. (CLEO), Baltimore, MD, Paper CThK1.

[10] A. L. Schawlow and C. H. Townes, “Infrared and optical masers,” *Phys. Rev.*, vol. 112, no. 6, pp. 1940–1949, Jun. 1958.

[11] S. Kobayashi, Y. Yamamoto, M. Ito, and T. Kimura, “Direct frequency modulation in AlGaAs semiconductor lasers,” *IEEE Trans. Microw. Theory Tech.*, vol. 82, no. 4, pp. 428–441, Apr. 1982.

[12] A. M. Yacomotti, L. Furfaro, X. Hachair, F. Pedaci, M. Giudici, J. Tredicce, J. Javaloyes, S. Balle, E. A. Viktorov, and P. Mandel, “Dynamics of multimode semiconductor lasers,” *Phys. Rev. A*, vol. 69, pp. 053816-1–053816-9, May 2004.