Measurement of the Linewidth Enhancement Factor of Semiconductor Lasers Based on the Optical Feedback Self-Mixing Effect

Yanguang Yu, Guido Giuliani, *Member, IEEE*, and Silvano Donati, *Fellow, IEEE*

***Abstract*—A new method for the measurement of the linewidth enhancement factor of semiconductor lasers is presented, based on the interferometric self-mixing effect. It is a fast and easy to perform method that does not require radio frequency nor optical spectrum measurements. A small fraction of the emitted light is backreflected into the laser cavity by a remote target driven by a sine waveform. The mixing of the returned and the lasing fields generates a modulation of the optical output power in the form of an interferometric waveform, with a shape that depends on the optical feedback strength and the linewidth enhancement factor α, according to the well-known Lang–Kobayashi theory. We show that the value of α can be retrieved from a simple measurement of two characteristic time intervals of the interferometric waveform. Experimental results obtained on different laser diodes show an accuracy of $\pm 6.5\%$.**

***Index Terms*—Linewidth enhancement factor, optical feedback, self-mixing interferometry, semiconductor laser (SL).**

I. Introduction

SEMICONDUCTOR lasers (SLs) exhibit a strong variation of refractive index and gain when the injected carrier density is changed. The parameter describing this dependence is called linewidth enhancement factor α [1] and it is defined as $\alpha = (\partial n_R/\partial N)/(\partial n_I/\partial N)$, where N, n_R, and n_I are, respectively, the carrier density and the real and imaginary part of the refractive index. The value of α is of great importance for many SL applications, as it characterizes the linewidth, the chirp, and the response to optical feedback [2].

Among the different methods that have been proposed to measure α [2], a broad classification can be made as follows: 1) methods relying on the direct measurement of the subthreshold optical spectrum as the injected current is varied [3]; 2) methods based on radio-frequency measurements [4]; and 3) techniques based on the analysis of the locking regimes induced by optical injection from a master laser [5], [6].

The present work describes a new method for the measurement of α, based on the interferometric self-mixing effect, that occurs when a small fraction of the light emitted by an SL is backreflected or backscattered by a remote target and it is allowed to reenter the SL cavity. The mixing of the lasing light with the backreflected light generates a slight variation of the carrier density, which causes a modulation of the emitted power in the form of an interferometric signal, that is a function of the phase of the backreflected light $\phi = 2k_0L$, where L is the distance of the target and k_0 is the wavenumber without optical feedback. The self-mixing technique has been previously applied to perform the measurement of metrological quantities [7], or SL parameters such as the linewidth [8]. According to the well-known Lang–Kobayashi theory for SLs with optical feedback [9], the resulting interferometric waveform depends on both the optical feedback strength and on the value of the linewidth enhancement factor α. In the present work, the value of α is evaluated by analyzing on an oscilloscope the shape of the self-mixing interferometric waveform in the moderate feedback regime as the target is put into vibration, exploiting the dependence of the shape of the waveform on the value of α. In particular, we have identified two features of the interferometric waveform that are easily measurable as time intervals on the oscilloscope, and that allow to retrieve the actual value of α, based on the Lang–Kobayashi theory. The proposed approach is interesting because of its inherent simplicity and compactness, as well as the self-aligning capability of the interferometric setup, that does not require mirror or corner cube reflectors.

II. Theory

Analytical solutions for the Lang–Kobayashi equations [9] for an SL with optical feedback can be found by solving the phase equation [10]

$$\omega_F(\tau)\cdot\tau = \omega_0\cdot\tau - C\cdot\sin\left[\omega_F(\tau)\cdot\tau + \operatorname{atan}\alpha\right] \quad (1)$$

where ω_0 is the angular frequency of the SL without feedback, ω_F is the angular frequency of the SL with feedback; $\tau = 2L/c$, with L the length of the external cavity and c the speed of light. The so-called feedback factor C is given by $C = \varepsilon((L\cdot\sqrt{1+\alpha^2})/(l\cdot n))\sqrt{R_{\text{ext}}}((1-R_2)/\sqrt{R_2})$, where R_2 is the power reflectivity of the SL output facet, R_{ext} is the reflectivity of the external target, l is SL cavity cavity length, n is SL cavity refractive index, and ε is an unknown coefficient that accounts for spatial mode overlap mismatch between the back-reflected light and the lasing mode (typ. $\varepsilon = 0.1 - 0.8$). Optical feedback phase changes the SL threshold condition, and the power emitted by the SL can, thus, be written as $P(\phi) = P_0[1 + m\cdot F(\phi)]$, where P_0 is the power emitted by the unperturbed SL, m is a modulation index (typ. $m \approx 10^{-3}$), and the interferometric function $F(\phi)$ is a periodic function of the

Manuscript received July 25, 2003; revised December 23, 2003. This work has been performed with partial support of FAR funding. One of the authors (Yanguang Yu) undertook this work with the support of the ICTP Program for Training and Research in Italian Laboratories, Trieste, Italy.

Y. Yu is with Dipartimento di Elettronica, Università di Pavia, Via Ferrata 1, I-27100, Pavia, Italy. She is on leave from the College of Information Engineering, Zhengzhou University, Zhengzhou, China (e-mail: yanguangyu@zzu.edu.cn).

G. Giuliani and S. Donati are with Dipartimento di Elettronica, Università di Pavia, Via Ferrata 1, I-27100 Pavia, Italy (e-mail: guido.giuliani@unipv.it).

Digital Object Identifier 10.1109/LPT.2004.824631

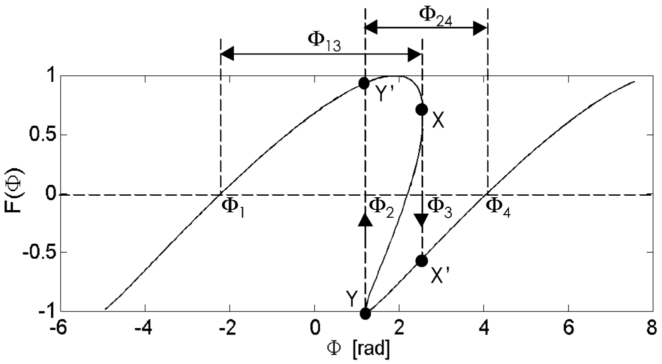

Fig. 1. Calculated plot of the function $F(\phi)$, representing the relative variation of the power emitted by the SL with feedback as a function of the interferometric phase ϕ (used values for the simulation are $C = 2$ and $\alpha = 3$).

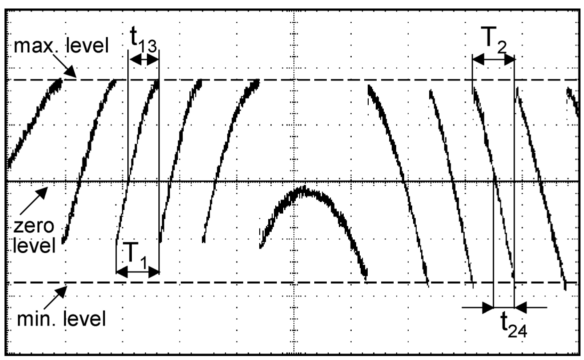

Fig. 2. Experimental self-mixing interferometric waveform obtained for moderate optical feedback from the monitor photodiode as the target is put into sine vibration with $\approx$2-μm peak-to-peak amplitude. The waveform is sawtooth-like and it exhibits hysteresis. Time intervals t_{13} and t_{24} relevant for the determination of α and C are shown on the graph. Horizontal scale: 2 ms/div. Vertical scale: 50 mV/div.

phase $\phi = \omega_0 \tau = 2k_0 L$ of period 2π. The function $F(\phi)$ takes the form [7], [10]

$$F(\phi) = \cos\left[\omega_F(\tau)\cdot\tau\right] = \cos\left[\omega_F(\phi)\cdot\frac{\phi}{\omega_0}\right]. \tag{2}$$

The feedback parameter C depends on both the optical feedback strength and external cavity length, and its value is useful to discriminate between different feedback regimes. For $1 < C < 4.6$, we have the moderate feedback regime, where the interferometric waveform $F(\phi)$ is approximately sawtooth-like and it exhibits hysteresis. As an example, Fig. 1 reports a calculated plot of $F(\phi)$ for $C = 2$ and $\alpha = 3$. Within each period, there are two points with infinite slope (X and Y), and the stability analysis shows that the branch between these two points is unstable. It is interesting to compare the theoretical plot with the experimental self-mixing signal reported in Fig. 2, obtained from the monitor photodiode included in the SL package as the external target is put into sine vibration. When the system is in point Y' and the interferometric phase is increased, the point moves along the curve up to point X, where it jumps down to point X'. Conversely, if the system is in X' and the phase is decreased, point Y is reached, and subsequentely an upper jump to point Y' occurs. Thus, when $C > 1$, the interferometric signal is discontinuous, exhibiting step-like transitions each time a 2π phase variation occurs (corresponding to $\lambda_0/2$ target displacement).

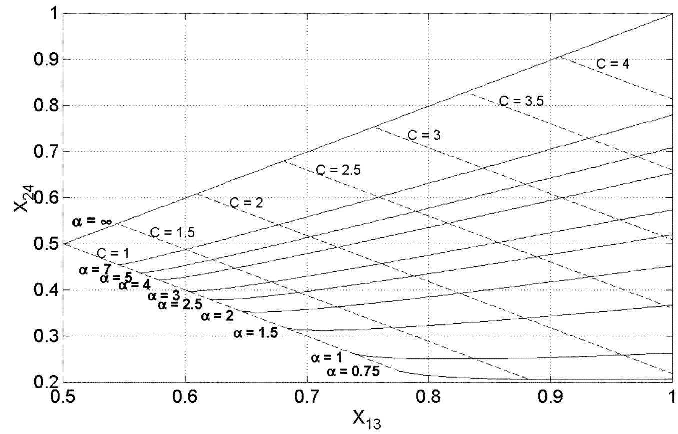

Fig. 3. Calculated contour plots in the $X_{24} - X_{13}$ plane corresponding to constant linewidth enhancement factor α (solid lines) and constant optical feedback factor C (dashed lines). Values for α and C are shown on the curves. The axes variables are defined as follows (see Fig. 1): $X_{13} = \phi_{13}/2\pi$, $X_{24} = \phi_{24}/2\pi$.

For our purpose, let us call ϕ_1 and ϕ_4 the phase values corresponding to a zero-crossing of the function $F(\phi)$, and ϕ_2 and ϕ_3, the phase values corresponding to points of $F(\phi)$ with infinite slope. Then, by solving (1) and (2), we can determine analytical expressions for the length of the segments ϕ_{13} and ϕ_{24}, also shown in Fig. 1

$$\phi_{13} = \sqrt{C^2-1} + \frac{C}{\sqrt{1+\alpha^2}} + \arccos\left(-\frac{1}{C}\right) - \arctan(\alpha) + \frac{\pi}{2} \tag{3a}$$

$$\phi_{24} = \sqrt{C^2-1} - \frac{C}{\sqrt{1+\alpha^2}} + \arccos\left(-\frac{1}{C}\right) + \arctan(\alpha) - \frac{\pi}{2}. \tag{3b}$$

From (3), the following adimensional quantities are derived: $X_{13} = \phi_{13}/2\pi$, $X_{24} = \phi_{24}/2\pi$, which can be easily measured experimentally. Fig. 3 reports calculated contour lines in the $X_{24} - X_{13}$ plane corresponding to constant values for the linewidth enhancement factor α and the feedback coefficient C. The knowledge of X_{13} and X_{24} leads univocally to the determination of α and C, either by the inverse solution of the set (3), or by graphical analysis carried out with the aid of Fig. 3. Interestingly, we note that the loci corresponding to $\alpha = \infty$ and $C = 1$ are described, respectively, by the equations $X_{24} = X_{13}$ and $X_{24} = -X_{13} + 1$.

III. Experiment

The interferometric self-mixing experimental setup used for the measurement of α is shown in Fig. 4. The SL is biased with a dc current, a microscope objective focuses light on the target (that is either a piece of white paper or Scotchlite retroreflector), and a variable attenuator is used to control the optical feedback. The target is mounted on a loundspeaker (or a piezoelectric transducer) driven by a sine signal at 40-Hz frequency. The self-mixing signal is obtained from the monitor photodiode connected to a transimpedance amplifier and recorded by a digital oscilloscope. An optical spectrum analyzer can also be used to check that the optical spectrum remains clean, with no mode-hopping and sidemode suppression of at least 10 dB,

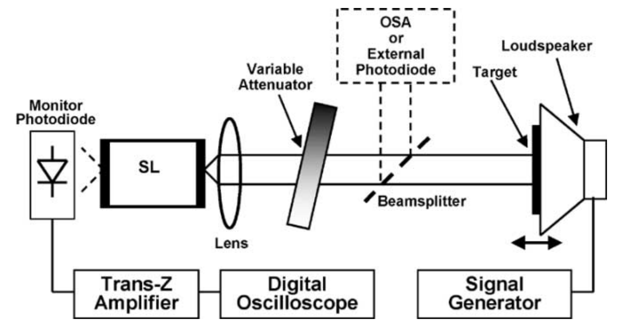

Fig. 4. Experimental setup for the measurement of linewidth enhancement factor in the optical feedback self-mixing interferometric configuration. The beam splitter can be used either to monitor the optical spectrum via an Optical Spectrum Analyzer (OSA), or to detect the signal with an external photodiode when using an SL without monitor photodiode.

TABLE I
SUMMARY OF MEASURED LINEWIDTH ENHANCEMENT FACTOR VALUES FOR DIFFERENT SL SPECIMEN. AT LEAST TEN EXPERIMENTAL MEASUREMENT DATA ARE TAKEN FOR EACH SL. ACCURACY IS CALCULATED AS STANDARD DEVIATION OF MEASURED DATA

Manufacturer	Model	Type	α	accuracy
SDL	SDL-7511-G1	635 nm, DFB	2.2	±8.5%
SANYO	DL7140-201	785 nm, F–P	3.1	±4.9%
HITACHI	HL8325G device #1	820 nm, F–P	3.2	±5.5%
HITACHI	HL8325G device #2	820 nm, F–P	3.4	±7%
MITSUBISHI	ML776H11F	1550 nm, DFB	3.1	±8%
MITSUBISHI	ML925B11F	1550 nm, DFB	4.9	±5%

which is required to meet the single-mode approximation of the Lang–Kobayashi theory.

A typical experimental self-mixing waveform is shown in Fig. 2, where we have indicated the time intervals t_{13} and t_{24} that allow us to determine X_{13} and X_{24} from $X_{13} = t_{13}/T_1$ and $X_{24} = t_{24}/T_2$, where T_1 and T_2 are the measured periods of a complete interferometric fringe. The measurement shall be performed where the sine drive signal is approximately linear, and the zero level is determined as the mean between the maximum and minimum values of the waveform.

Subsequent repeated measurements have been carried out for different LD specimen by varying the optical feedback strength, thus, obtaining a set of experimental data for each device. From these data, the average measured value for α has been obtained, together with the uncertainty. Among different specimen of the same type, a remarkable consistency of measured results was found. Some of the measured data are summarized in Table I.

Fig. 5 reports measured data plotted on the $X_{24} - X_{13}$ plane for three different LDs as obtained by varying the optical feedback strength. A good agreement is found with the extrapolated α values, for which theoretical curves are also plotted.

IV. CONCLUSION

We have presented a new method to determine the linewidth enhancement factor of an SL. This technique relies on the Lang–Kobayashi theory for SLs with optical feedback, and it can be implemented with a very simple experimental setup. The method can be applied to all single-mode SLs, and its accuracy

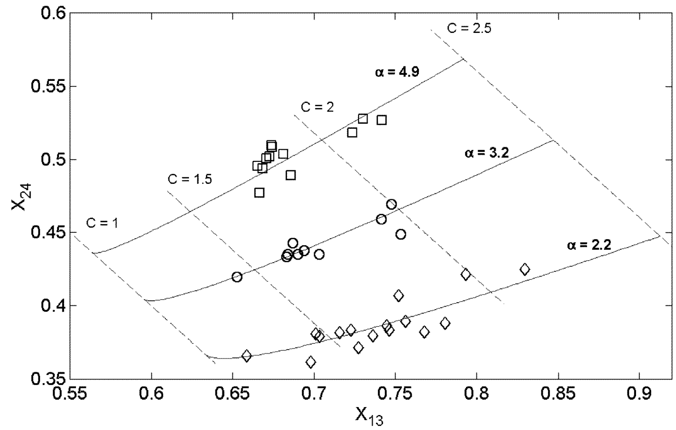

Fig. 5. Experimental data points plotted in the $X_{24} - X_{13}$ plane. The points are obtained from repeated measurements of time intervals t_{13} and t_{24} (see Fig. 2) for varying optical feedback strength. Axes variables are defined as: $X_{13} = t_{13}/T_1$, $X_{24} = t_{24}/T_2$. Squares: Mitsubishi ML925B11F, 1550 nm, DFB; estimated $\alpha = 4.9$. Circles: Hitachi HL8325G#1, 820 nm, Fabry–Pérot; estimated $\alpha = 3.2$. Diamonds: SDL SDL-7511-G1, 635 nm, DFB; estimated $\alpha = 2.2$. A good agreement between measured data and the estimation is found. Contour lines for constant C values are also plotted.

is estimated ±6.5%. Compared to other methods based on optical feedback, the proposed approach has the advantage that it does not require the measurement of the feedback strength, which cannot in general be determined with good accuracy. Moreover, this approach can be also useful for a measure of the effective feedback strength.

ACKNOWLEDGMENT

Y. Yu would like to thank A. Fanzio for technical assistance during measurements

REFERENCES

[1] C. H. Henry, "Theory of the linewidth of semiconductor lasers," *IEEE J. Quantum Electron.*, vol. QE-18, pp. 259–264, Feb. 1982.

[2] M. Osinski and J. Buus, "Linewidth broadening factor in semiconductor lasers—an overview," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 9–28, Jan. 1987.

[3] I. D. Henning and J. V. Collins, "Measurement of the linewidth broadening factor α of semiconductor lasers," *Electron. Lett.*, vol. 19, pp. 927–929, 1987.

[4] K. Kikuchi and T. Okoshi, "Estimation of linewidth enhancement factor of AlGaAs lasers by correlation measurement between FM and AM noises," *IEEE J. Quantum Electron.*, vol. QE-21, pp. 669–673, June 1985.

[5] G. Liu, X. Jin, and S. L. Chuang, "Measurement of linewidth enhancement factor of semiconductor lasers using an injection-locking technique," *IEEE Photon. Technol. Lett.*, vol. 13, pp. 430–432, May 2001.

[6] K. E. Chlouverakis, K. M. Al-Aswad, I. D. Henning, and M. J. Adams, "Determining laser linewidth parameter from Hopf bifurcation minimum in lasers subject to optical injection," *Electron. Lett.*, vol. 39, pp. 1185–1187, 2003.

[7] G. Giuliani, M. Norgia, S. Donati, and T. Bosch, "Laser diode self-mixing technique for sensing applications," *J. Opt. A: Pure Appl. Opt.*, vol. 4, no. 6, pp. S283–S294, 2002.

[8] G. Giuliani and M. Norgia, "Laser diode linewidth measurement by means of self-mixing interferometry," *IEEE Photon. Technol. Lett.*, vol. 12, pp. 1028–1030, Aug. 2000.

[9] R. Lang and K. Kobayashi, "External optical feedback effects on semiconductor injection laser properties," *IEEE J. Quantum Electron.*, vol. QE-16, pp. 347–355, Mar. 1980.

[10] K. Petermann, *Laser Diode Modulation and Noise*. Dordrecht, The Netherlands: Kluwer, 1988.