# Inductors and Transformers

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#### Magnetic Devices

• Inductors: devices used to store and release energy in magnetic form.

• <u>Transformers</u>: devices used to transfer power instantaneously from one circuit (primary) to another (secondary and vice-versa. Energy storage is an undesired parasitic effect.

#### Inductors

- An inductor is usually constructed as a coil of conducting material, wrapped around a core either of air or of ferromagnetic material.
- Core materials with large magnetic permeability ( $\mu_R$ >>1) confine the magnetic field closely to the inductor, thereby increasing the inductance



Power



RF

Contraction of the second second

EMI

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#### Transformers

• The fundamental purpose of magnetic cores in transformers is to facilitate magnetic flux linkage (coupling) between primary and secondary transformer windings





#### Outline

#### **EM basics: Faraday and Ampere**

#### Wire losses

 Resistive losses in the windings: DC resistance and skin effect

#### Ferromagnetic core losses

• Hysteresis and saturation losses

#### Eddy currents

• <u>Resistive losses</u> in the core due to magnetically induced eddy currents

#### Materials

## **Magnetics Basics**

#### FARADAY'S LAW

$$v(t) = \frac{\partial \Phi_B}{\partial t}$$



# $MMF = \oint_{ClosedPath} H \cdot dl = NI_{Int.toPath}$



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#### Self Inductance



- A simple inductor can be built with a single loop of wire made of a good conductor
- Calculating self inductance L is not trivial





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R

# Low Frequency Conductor Loss

- Low frequency conductor loss can be modeled as a resistor in series.
- Loss resistance is inversely proportional to wire cross section A<sub>W</sub>.
- Copper resistivity is 1.7 10<sup>-6</sup> Ω·cm at room temperature

• Power loss is 
$$P_{diss} = \frac{1}{2}RI^2$$

☐ The inductor quality factor (Q) is equal to the ratio of the stored magnetic energy to the energy dissipated in one cycle.

$$Q = \frac{E_m}{\int_T P_{diss}} = 2\pi \frac{1/2 LI^2}{1/2 RI^2 T} = \frac{\omega L}{R} \approx \left(\ln \frac{r}{a}\right) \frac{\pi f \mu_0 \mu_r a^2}{\rho_{Cu}}$$

$$R = \rho_{cu} \frac{l}{A_w} \cong \rho_{cu} \frac{2r}{a^2}$$

#### r loop Radius a wire radius

### High Freq. Losses: Skin Effect



- At high frequencies the current flow concentrates on the outer surface of the wire
- Current density decreases exponentially with a characteristic length equal to the skin depth  $\delta$
- Resistive losses increase at high frequencies

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$
For copper at room  
temperature:  
$$\delta = \frac{7.5}{\sqrt{f}} \text{ cm}$$

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#### Magnetic Materials

 $B = \mu_0 \mu_R H$ 

$$\mu_R = 1 + \chi_M$$

 Diamagnetic atoms/molecules without magnetic dipole Larmor precession, Larmor momentum (μ<sub>r</sub> ~1)
 Paramagnetic atoms/molecules with magnetic dipole (μ<sub>r</sub> ~ 1)
 Ferromagnetic atoms/molecules dipole exhibiting strong intrinsic magnetization (μ<sub>r</sub>>>1)

# **Magnetic Properties of Materials**

#### Paramagnetic material ( $\mu = \mu_0 \mu_R$ ; $\mu_R \sim 1$ ) Ferromagnetic material



- $\mu_0$  = magnetic permeability in free space (  $4\pi$  10<sup>-7</sup> H/m)
- B<sub>SAT</sub> saturation flux density
- B<sub>R</sub> retention flux density
- H<sub>c</sub> coercion field

### **Ferromagnetic Materials**

- Weiss domains are local regions inside the material (few μm to a 100μm) with strong intrinsic magnetization
- Atomic-level magnetic dipoles *m* are subject to orientation by the local magnetic field H<sub>L</sub> giving rise to an hysteresis cycle



•  $H_L$  is a strong function of global magnetization M (  $\gamma \sim 10^3$  )

 $(1) H_L = H + \gamma M$ 

• Magnetization is a function of local magnetic flux density (  $B_L = \mu_0 H_L$ ) through the Langevin function

(2) 
$$M = M_{S} \left( \operatorname{coth} \left( \frac{m\mu_{0}H_{L}}{KT} \right) - \frac{KT}{m\mu_{0}H_{L}} \right) \right)$$

M T T BL

### Ferromagnetic Materials (2)

- H<sub>L</sub> is a function of the applied magnetic field (H) and of local magnetization M
- Local magnetization is a function of the local magnetic flux density  $B_L = \mu_0 H_L$
- M (and hence B) can be found as a function of H by combining the two relationships
- When there is hysteresis more than one solution is expected
- Let us try to do this graphically

#### **Hysteresis**



We find the M-H (B-H) relationship graphically by choosing H and finding the value of M that satisfies the equations.

As H is increased M also increases, but, as expected, for certain values of H more than one value of M is possible: Hysteresis!

#### **Hysteresis and Curie Temperature**

Now we raise the temperature (above the Curie temperature  $T_C$ ) and repeat the experiment.

Now the slope of the Langevin function is smaller

One value of M is possible for each value of H: No more hysteresis!



#### Magnetic Core

- The fundamental purpose of magnetic cores is to provide a "low impedance" path for the magnetic flux
- This is a direct result of boundary continuity conditions:



#### Summary

- Ferromagnetic materials exhibit large magnetic permeability with hysteresis cycle
- The ferromagnetic property is sensitive to temperature:  $\mu_R$  decreases with temperature and above the Curie temperature the hysteresis disappears
- The ferromagnetic material acts as a "low impedance" path for the magnetic flux

# INDUCTORS WITH MAGNETIC CORE

# Inductors with Magnetic Core

#### FARADAY'S LAW

#### For each turn:

$$v_{turn} = \frac{\partial \Phi_B}{\partial t}$$

Total voltage:

$$V = N \frac{\partial \Phi_B}{\partial t}$$

Assuming constant flux density:

$$\Phi_B = BA$$

$$V = NA \frac{\partial B}{\partial t}$$



#### AMPERE'S LAW

Assuming constant magnetic field:

$$Hl_m = NI$$
  $\longrightarrow$   $H = \frac{NI}{l_m}$ 

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#### Self Inductance



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#### Inductor w/ Magnetic Core

$$L = \mu N^2 \frac{A}{l_m}$$

Advantage✓ High inductance

Issues:

- Temperature dependence
- Saturation
- Hysteresis losses

#### **Saturation Current**





- Saturation occurs when flux density exceeds B<sub>SAT</sub>
- For  $|I| > I_{SAT}$  the flux density is constant and equal to  $B_{SAT}$ .
- Faraday's law then predicts:  $V = NA \frac{dB}{dt} = 0$
- In saturation the inductor becomes a short circuit!

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# Hysteresis Energy Loss



# Hysteresis Energy Loss

- SUPPLIED ENERGY RELEASED ENERGY B Ρ н H P<sub>1</sub>
- The amount of energy that is released is always less than what is supplied
- Materials having high retention flux (B<sub>R</sub>~B<sub>SAT</sub>) have low recoverable energy

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# **Hysteresis Energy Loss**

- When a full hysteresis cycle is over the amount of energy corresponding to the shaded area is lost
- Energy loss per cycle is constant, independent of the duration of the cycle (signal frequency)
- Power loss is proportional to the duration of the cycle (signal frequency)



#### Hysteresis Loss



 $V = NA \frac{\partial B}{\partial t}$ 

$$\checkmark Vdt = NAdB$$
$$\checkmark I = \frac{Hl_m}{N}$$

$$Hl_m = NI$$

Energy lost per cycle:

$$\int_{cycle} VIdt = (Al_m) \int_{cycle} HdB$$





- A good inductor is made with non-retentive materials
  - If B<sub>R</sub> is close to B<sub>SAT</sub> less energy is recovered
- Q is constant over frequency

#### Summary

- A large inductance is obtained using a multiplewindings inductor and a ferromagnetic core
- A good inductor is made with non-retentive materials
  - > If  $B_R$  is close to  $B_{SAT}$  more energy is lost
- Each hysteresis cycle leads to a fixed energy loss: power loss is directly proportional to frequency

# Eddy Current Loss



Flux variation within the core induces a voltage (Faraday's law) within the core itself.

**Magnetic core** materials have finite **resistivity**, hence currents flow within the core ("eddy currents").

Eddy currents generate a magnetic flux that, according to Lenz's law, counteracts the (main) core flux  $\Phi$ , and induce power losses.

If the magnetic core material is a good conductor, eddy current losses can be substantial.

# **Eddy Current Loss**

- For a given magnetic energy stored (½ Ll<sup>2</sup>), hence for a given current, voltage increases with frequency
  - Power dissipated due to eddy currents raises as f<sup>2</sup>
- Eddy current losses can be modeled as a **resistor in parallel** to the inductance (resistance is independent of frequency)
- Eddy currents induce a quality factor which is inversely proportional to frequency

$$P_{LOSS} \propto rac{V_{core}^2}{
ho} \propto rac{f^2}{
ho}$$

$$Q = \frac{E_m}{\int_T P_{diss}} = 2\pi \frac{1/2 L I^2 R_p}{1/2 \omega^2 L^2 I^2 T} \approx \frac{R_p}{\omega L}$$

#### **Laminated Cores**

- In order to reduce eddy currents, the core can be laminated, i.e. divided into several insulated thinner layers
- Power loss decreases with the square of the number of insulated laminations (N)



#### Inductors with Air Gap



• What is the value of the inductance?

• What are the parameters influencing the "quality" of the inductor?

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#### Magnetic Circuits



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# Magnetic Circuits



### **Equivalent Circuit Elements**



Equivalent circuit rules:

- Each magnetic branch substituted with a reluctance
- MMF substituted with equivalent voltage sources
- Magnetic flux as current
- Solve equivalent circuit using Kirchoff's laws

#### **Equivalent of KCLs**

Total MMF around a closed path (including winding currents) adds up to zero.

$$\sum MMF_i = 0$$

• Since 
$$\nabla \cdot B = 0$$
 total magnetic

flux entering a node must be zero.

$$\begin{array}{c|c} \Phi_1 & \Phi_3 \\ \hline & \Phi_2 \end{array}$$

$$\Phi_1 + \Phi_2 = \Phi_3$$

$$\sum \Phi_i = 0$$

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Φ

 $\frac{NI}{\Re_{core} + \Re_{gap}}$ 

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### Inductor with Air Gap



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#### Inductor with Air Gap

$$\Phi = \frac{NI}{\Re_{core} + \Re_{gap}}$$

$$V = N \frac{d\Phi}{dt} = \frac{N^2}{\Re_{core} + \Re_{gap}} \frac{dI}{dt}$$

$$L = \frac{N^2}{\Re_{core} + \Re_{gap}} \cong \frac{\mu_0 A N^2}{l_g}$$

Effect of air gap:

- Inductance less dependent on core permeability:
  - Smaller inductance (but still much larger than with air core)
  - Reduced sensitivity to temperature (temperature)

Is saturation still there?

# Effect of Air Gap: Simplified Model

#### in gap.

- sed saturation current
- ased maximum stored energy

$$E_{M} = \frac{1}{2} \Phi^{2} \left( \Re_{core} + \Re_{gap} \right)$$

$$L = \frac{N^2}{\Re}$$

$$I/(\mathcal{R}_{core}, \mathcal{R}_{gap})$$

$$I_{SAT}$$



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An air gap inside a magnetic circuit stores magnetic energy. Compared to air-core inductors, self-inductance is increased. Saturation current and maximum stored energy is higher than without air core.

# **TRANSFORMERS**

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#### Transformers

• The fundamental purpose of magnetic cores in transformers is to facilitate magnetic flux linkage (coupling) between primary and secondary transformer windings



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# Equivalent Circuit (Ideal)

#### Equivalent Magnetic Circuit

#### **Equivalent Electrical Circuit**



Ideal transformer approximation:

$$\Re_{core} = \frac{l_m}{\mu A} \xrightarrow{\mu \to \infty} 0$$

$$N_1I_1 + N_2I_2 = \Phi \Re \rightarrow 0$$

$$I_1 N_1 = -I_2 N_2$$

$$\mathbf{V}_1 \left( \begin{array}{c} \mathbf{I}_1 \\ \mathbf{N}_1 \\ \mathbf{N}_1 \\ \mathbf{N}_2 \\ \mathbf{I}_2 \\ \mathbf{V}_2 \end{array} \right) \mathbf{V}_2$$

Applying Faraday's law:

$$V_1 = N_1 \frac{d\Phi}{dt} \qquad V_2 = N_2 \frac{d\Phi}{dt}$$

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

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#### **Transformer Magnetization**

In a real transformer winding currents ratio differs from the turn ratio:

$$N_1I_1 + N_2I_2 = \Phi \Re = I_M$$
 Magnetizing current

Magnetizing current ( due to finite core reluctance  $\mathcal{R}$  ) leads to magnetic energy storage:

$$\begin{split} \mathbf{E}_{M} &= \frac{1}{2} L_{1} I_{1}^{2} + \frac{1}{2} L_{2} I_{2}^{2} + M I_{1} I_{2} \\ &= \frac{1}{2} \frac{M}{N_{1} N_{2}} (N_{1}^{2} I_{1}^{2} + N_{2}^{2} I_{2}^{2} + 2N_{1} N_{2} I_{2} I_{1}) \\ \mathbf{E}_{M} &= \frac{1}{2} \frac{M}{N_{1} N_{2}} I_{M}^{2} = \frac{1}{2} \frac{I_{M}^{2}}{2 \Re} = \frac{1}{2} \Re \Phi^{2} \\ I_{M} &= \frac{\Re}{N_{1}} \int V_{1} dt \\ &\Re = \frac{l_{m}}{\mu A} \end{split}$$

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#### **Transformer Non-idealities**

In a real transformer core reluctance  $\mathcal{R}$  is finite:



#### **Real Transformer Model**



the ratio between the current on the primary and secondary is different from the turns ratio  $N_2/N_1$ 

#### **Transformer Saturation**

- Saturation occurs when magnetic flux density exceeds B<sub>SAT</sub>
- When the core saturates the power transfer mechanism is blocked
- Large winding currents do not necessarily lead to saturation if  $N_1I_1 + N_2I_2 \approx 0$
- Saturation is caused by excessive integral applied voltage

$$B = \frac{1}{N_1 A} \int V_1 dt$$

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# Filter (Switched Power Supply)





$$H_c(t) = \frac{ni(t)}{\ell_c} \frac{\mathcal{R}_c}{\mathcal{R}_c + \mathcal{R}_g}$$

- Air gap is used to store energy
- Loss dominated by wire loss
  - Negligible core loss
- Flux density chosen to avoid saturation

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#### AC Transformer



#### Hysteresis loss (prop. to freq.)

# MATERIALS

## Silicon Steel

- Iron with a small percentage (1-4%) of Silicon and Carbon. The addition of silicon reduces hysteresis loss at the price of increased brittleness and lower B<sub>SAT</sub>
- Due to very low resistivity (ρ~50μΩ·cm), to reduce losses induced by eddy currents, these cores are made in thin (~100μm) laminations. Electrical isolation is obtained by oxidation.
- Saturates at about 2.1T
- **Grain oriented silicon steel** is achieved by rapidly cooling (quenching) the material (Fe crystals become oriented), resulting in increased magnetic permeability ( $\mu_r$  ~40.000) and reduced hysteresis losses
- Losses due to hysteresis and eddy currents are moderately high
- Applications: low-frequency, high-power transformers, audio frequencies (grain oriented)

## Metal Alloys

- Metal alloys of iron and nickel (Fe-Ni) or cobalt (e.g. Permalloy) have high magnetic permeability ( $\mu_r \sim 60.000$ ), high saturation flux density (0.9T) and narrow hysteresis cycle
- Resistivity is low (~100 $\mu\Omega$ ·cm) leading to strong eddy currents
  - To reduces losses induced by eddy currents these cores are made in thin (~10µm) tape-wound laminations
- □ Main application is in low frequency (50-60Hz) transformers

#### **Powdered Metal Cores**

- Composite materials made of a non-magnetic binder that holds together powdered metals such as iron and Permalloy.
  - Non-magnetic inclusions store significant energy within the core, acting as a "distributed gap"
- "Effective" permeability is low and varies depending on the composition:  $\mu_r$  in the range 10-200
- Saturation flux density is intermediate (0.6-0.8T)
- Main application is in inductors for intermediate frequencies (~100kHz)

#### **Ferrites**

- Ferrites are usually non-conductive ferrimagnetic ceramic compounds derived from iron oxides such as hematite (Fe<sub>2</sub>O<sub>3</sub>) or magnetite (FeO-Fe<sub>2</sub>O<sub>3</sub>) as well as oxides of other metals.
   "Soft" ferrites obtained using nickel, zinc, or manganese.
- **Resistivity is high**, in the range of 200-2k  $\Omega$ -cm
- Magnetic permeability is high:  $\mu_r \sim 1500-3000$ .
  - For high frequency (>100MHz) applications Ni-Zn are used with  $\mu_r \sim 20$
- Narrow hysteresis cycle
- Saturation flux density is low (~0.3T)
- Applications: inductors, RF transformer, switched-mode power supplies
  - They are commonly seen as a lump in a computer cable, called a ferrite bead, which helps to prevent high frequency (RF )interference) from exiting or entering the equipment.



#### **Ferrites**

#### "Hard" ferrites

- Permanent ferrite magnets which have a high remanence after magnetization, are composed of iron and barium or strontium oxides.
- In a magnetically saturated state they conduct magnetic flux well and have a high magnetic permeability. This enables these socalled ceramic magnets to store stronger magnetic fields than iron itself.
- The saturation flux density is low: about 0.3 T
- Applications: Loudspeakers. They are the most commonly used magnets in radios.

## **Core Materials Summary**

Core type	$B_{sat}$	Relative core loss	Applications
Laminations iron, silicon steel	1.5 - 2.0 T	high	50-60 Hz transformers, inductors
Powdered cores powdered iron, molypermalloy	0.6 - 0.8 T	medium	1 kHz transformers, 100 kHz filter inductors
Ferrite Manganese-zinc, Nickel-zinc	0.25 - 0.5 T	low	20 kHz - 1 MHz transformers, ac inductors

#### Summary

- Magnetic devices can be modeled using lumped-element magnetic circuits, similar to electrical circuits.
- The magnetic analogs of electrical voltage V, current I, and resistance R, are magnetomotive force (MMF) F, flux  $\Phi$ , and reluctance  $\Re$  respectively.
- Faraday's law relates the voltage induced in a loop of wire to the derivative of flux passing through the interior of the loop.
- Ampere's law relates the total MMF around a loop to the total current passing through the center of the loop. Ampere's law implies that winding currents are sources of MMF, and that when these sources are included, then the net MMF around a closed path is equal to zero.

# Summary (2)

- Magnetic core materials exhibit hysteresis and saturation. A core material saturates when the flux density *B* reaches the saturation flux density B<sub>SAT</sub>.
- Core loss are due to hysteresis of the *B*–*H* loop and to induced eddy currents flowing in the core material.
- Air gaps are employed in inductors to prevent saturation and to stabilize the value of inductance. The inductor with air gap can be analyzed using a simple magnetic equivalent circuit, containing core and air gap reluctances and a source representing the winding MMF. The gap reluctance dominates, determining the inductance value.

# Summary (3)

- Conventional transformers can be modeled using sources representing the MMFs of each winding, and the core MMF. The core reluctance approaches zero in an ideal transformer (no magnetic energy storage).
- Nonzero core reluctance leads to an electrical transformer model containing a magnetizing inductance, effectively in parallel with the ideal transformer (that stores magnetic energy).
- The conventional transformer saturates when the applied winding volt-seconds are too large. Addition of an air gap has no effect on the magnetic flux density saturation.
- Saturation can be prevented by increasing the core cross-sectional area, or by increasing the number of primary turns.

### Summary (4)

- In available core materials, there is a tradeoff between high saturation flux density  $B_{SAT}$  and high core loss  $P_{DISS}$ . Laminated iron alloy cores exhibit the highest  $B_{SAT}$  but also the highest  $P_{DISS}$ , while ferrite cores exhibit the lowest  $P_{DISS}$  but also the lowest  $B_{SAT}$ . Between these two extremes are powdered iron alloy and amorphous alloy materials.
- The skin effect leads to eddy currents in winding conductors, which increase the copper loss in high-current high-frequency magnetic devices.

#### References

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