Interferometric Measurements of Displacement on a Diffusing Target by a Speckle Tracking Technique

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***Abstract*—Operation of a laser interferometer with a noncooperative target surface, and on a substantial span of displacement (1 m), is reported for the first time to our knowledge. In the paper, we first analyze the errors of an interferometer operating in the speckle pattern regime, then propose the technique of tracking the speckle relative maximum amplitude as a solution for problems so far preventing interferometric measurements of large displacements. Examples of the speckle-statistics improvements are reported. Residual errors are of the order of few wavelengths on a 1-m displacement, indicating that a 10^{-6}-class instrument can be developed. The proposed approach has been implemented with an injection interferometer.**

***Index Terms*—Laser measurements, optical feedback, optical interferometry, rough surfaces, semiconductor lasers, speckle.**

I. Introduction

SHORTLY after the advent of the first He–Ne frequency-stabilized lasers in 1962, laser interferometers for displacement measurements with sub-micrometer resolution, multimeter dynamic range and 10^{-6} accuracy have been developed, the first being perhaps the HP 5526 instrument which appeared in 1965. Since then, laser interferometers have become widespread in optical and tool–machine workshops, and can today be regarded as one of the most remarkable technical and industrial successes of electrooptical measurement science [1].

Though largely accepted in practical use, some hindrance still remains in the operation of a laser interferometer. First, it supplies an incremental measurement of the displacement and, accordingly, if some countings are lost because of a transient signal dropout, the measurement is incorrect.

Second, and the aim of this work, the operation of the interferometer requires a reflective target. Usually, the target is a corner-cube mounted on the moving object under test so as to configure a Twyman–Green optical interferometer, thereby alleviating somewhat the alignment criticality. Though a corner-cube is reluctantly accepted after all, it would be much better to be able to work directly with a diffuser surface as found in the normal workshop environment, with no invasiveness or the need to keep optical surfaces clean.

The situation is quite different from that of vibration-sensing interferometers, already demonstrated to detect the very small (nanometer to picometer) amplitude of vibrations on diffusers in a number of scientific and technical applications, ranging from SAW detection to biological motility to large-structure integrity. Vibration-sensing is a special easy case of interferometry, where the dynamic range is very small and can be kept much less than the speckle longitudinal size. This means that the phase error is small, and amplitude fading is simply circumvented by changing the speckle on the target when we fall on one with a weak intensity.

In this paper, we first analyze the source of errors encountered in interferometric measurements. Then, we propose speckle maximum-amplitude tracking as a cure to amplitude fading, and illustrate the viability of the method by a number of examples. The performance of the system operating over a wide dynamic range (e.g., 0.5–1 m) with fraction-of-wavelength resolution, is reported.

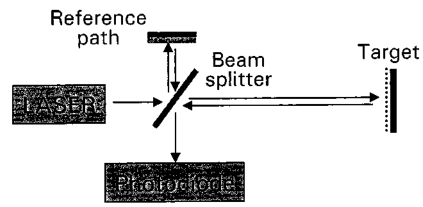

Fig. 1. Michelson interferometer.

II. Interferometry in the Speckle Regime

When attempting to make an interferometric measurement on a diffusing surface, problems in amplitude and phase are found. Let us briefly discuss this point, considering a Michelson scheme as a general case (Fig. 1).

A. Amplitude

First, the signal power returning to the beamsplitter combiner (Fig. 1) from a diffuser is reduced by a factor $N = A\,\Omega/\lambda^2$ with respect to the mirror target, where N is the number of spatial modes of the target, as given by the target area A and the radiating solid angle Ω ($\Omega = \pi$ for an ideal diffuser in a half-space) [2], [3], and λ is the laser wavelength. Only when the target is in the focal plane of a lens—the case of the vibrometer—can N be a moderately low number. In a displacement interferometer, we have $A = \pi[w_0^2 + (\lambda z/\pi w_0)^2]$ for a Gaussian beam, and at a distance up to z =1 m we may have $N = 10^4 - 10^6$ typically (w_0 is the focussed spot dimension).

However, the signal reduction only impacts the minimum-detectable displacement. Indeed, even with a very small signal

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power P/N, the interferometer is performing a coherent detection [3], because of the superposition of the reference beam. Thus, the quantum limit of P/N is always attained, and the signal-to-noise ratio is S/N= $\sqrt{[P/N2h\nu B]}$, where ν is the optical frequency, B the electrical bandwidth and h the Planck constant. The minimum-detectable-displacement is $\mathrm{MDD} = \lambda/2\pi$ (S/N) [3], therefore, with practical values (nm to pm) still satisfactory even for high ($10^4 - 10^6$) attenuations.

Second, and more of concern, the average power $\langle P \rangle = P/N$ is subjected to speckle-regime statistics, and has a probability density [2], [4]

$$p(P) = \frac{e^{-P/\langle p \rangle}}{\langle P \rangle}. \tag{1}$$

Small amplitude speckles are relatively frequent (e.g., 10% have less than 10% the average power); thus, signal can be lost by fading when moving longitudinally the target along speckles. Of course, an automatic gain control (AGC) of the received signal and/or the doubling of the measurement channel can help reducing the probability of fading below a specified level, but it will not completely eliminate the fading problem.

B. Phase

The signal field returning to the detector can be written as: $E = aE_0 \exp i(2kz + \varphi_{fc} + \varphi_{sp})$, where a is the amplitude attenuation term, $2kz$ is the desired optical phase-shift, $\varphi_{fc} = kw^2/2z$ is a field-curvature error term due to the finite extent w of the target as seen from a distance z, and φ_{sp} is the random phase error due to the speckle statistics [2], [4].

The quantitative error made in a displacement Δz of the target is $\Delta\varphi_{fc} = kw^2\Delta z/2z^2$ for the field curvature. This is a deterministic error we may make either small by working in the far-field, or correct by a first-order estimate of z.

Instead, the speckle error is a random one and cannot be corrected. The statistics of the speckle phase-difference $\Delta\varphi_{sp} = \varphi_1 - \varphi_2$ upon a displacement from a point P_1 to a point P_2 has been studied in [4].

One source of error is the longitudinal displacement itself, giving a term $\Delta\varphi_{sp} = 2\pi\sqrt{(\Delta z/s_z)}$, where $s_z = \lambda(z/w)^2$ is the longitudinal speckle size [2]. Again, we may reduce this error by working in the far-field.

A second, more serious source of error is that generated by a transversal displacement Δr of the beam on the target, given by $\Delta\varphi_{sp} = 2\pi\sqrt{(\Delta r/w)}$. In practical operation, as the target surface may undergo a transversal displacement during the measurement, this error can become very large.

III. Self-Mixing Interferometry

A. Basic Principles

In our experiments, we employ an interesting alternative to the classical interferometric configuration, that is, a laser diode self-mixing interferometry, also variously referred to as injection, retro-reflection, or feedback interferometer [5]–[8]. This configuration takes advantage of the amplitude modulation arising when a small fraction of the emitted light is reflected back into the laser cavity [5]–[8].

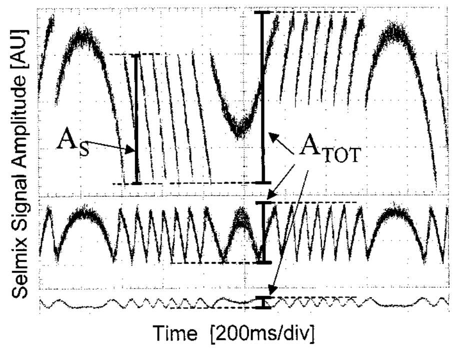


Fig. 2. Self-mixing signal at increasing levels (bottom to top) of back-injected power.

As an illustration, three examples of self-mixing signals obtained at different power levels of retro-reflection from the remote target into the cavity of a laser diode (we used Hitachi HL8325) are shown in Fig. 2. The signals are taken using a target sinusoidally vibrating, with an amplitude of 3 μm. The signal waveform depends on the feedback parameter C, which is proportional to the amplitude of the back-injected electric field and to the target distance z [5]

$$C = \frac{z}{\sqrt{A}} \cdot \frac{\varepsilon\sqrt{1+\alpha^2}}{L_{\mathrm{las}} n_{\mathrm{las}}} \cdot \frac{1 - R_{\mathrm{out}}}{\sqrt{R_{\mathrm{out}}}} \tag{2}$$

where

A	power attenuation along the optical path;
α	linewidth enhancement factor;
ε	mode mismatch coefficient (~0.5);
L_{las}	laser length;
n_{las}	refractive index;
R_{out}	reflection coefficient of the laser output facet.

For low injection levels ($C < 1$), the signal is nearly sinusoidal and it increases linearly in amplitude with an increasing level of the back injected field. When injection becomes strong ($C = 1$, for $A \sim 10^6$), the signal waveform becomes saw-tooth like. If the amount of injection is increased further ($C > 1$), the signal shows a hysteresis in the amplitude (upper trace in Fig. 2). In the latter case, we can measure the target displacement without ambiguity, simply by counting the sharp up/down transitions of the signal [5]. Each count represents a displacement equal to half-wavelength, and is to be taken positive or negative for an upward or downward transition, respectively.

B. Working on Diffusive Targets

Because of the speckle pattern regime, the $C > 1$ condition of strong-injection is harder to match. We can evaluate the probability of having $C > 1$ by studying the speckle distribution. While the power distribution is described by well-known negative-exponential statistics given by (1), in a self-mixing interferometer the signal is proportional to the amplitude $|E|$ of the field

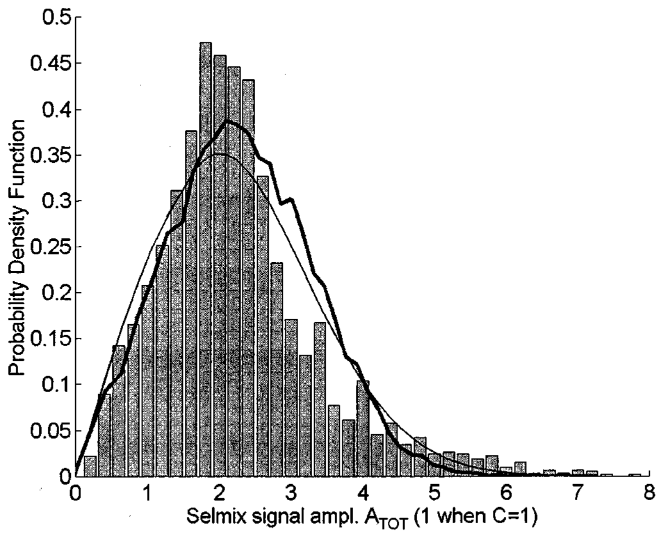


Fig. 3. PDF of the self-mixing signal amplitude when a spot is focussed on a target vibrating 50-cm away from the laser. Bars are experimental data. Thin line is the theoretical Rayleigh distribution for ideal diffusive target. Thick line is the result of a numerical simulation assuming 98% diffusion and 2% reflection from the target.

back injected, (like in an usual coherent detection). The probability density function (pdf) of the amplitude $|E|$ is a Rayleigh distribution and can be written [2] as

$$p(|E|) = p\left(\sqrt{P}\right) = p(P)\frac{dP}{d\sqrt{P}} = 2\sqrt{P}p(P) = \frac{2|E|}{\langle P\rangle} e^{-|E|^2/\langle P\rangle}. \quad (3)$$

The self-mixing signal amplitude ($A_{\rm TOT}$ in Fig. 2) dependence on $|E|$ is linear only for moderate injection, or $C < 1$, whereas for strong injection it exhibits saturation. An approximate relation has been found experimentally to fit fairly well; that is

$$A_{TOT} \propto \begin{cases} |E|, & \text{for } C \leq 1.4 \\ |E_1| + (|E| - |E_1|) \cdot 0.7, & \text{for } C > 1.4 \end{cases} \quad (4)$$

where E_1 is the injected electric field that gives $C = 1.4$.

Fig. 3 shows the experimental PDF of the self-mixing amplitude signal $A_{\rm TOT}$. The signal is obtained by the measurement on a white-paper target, put into vibration with an amplitude of a few micrometers, at a 50-cm distance of from the laser. The graph is made with 3000 samples, taken by changing the spot position on the surface. Other curves in Fig. 3 show the theoretical Rayleigh distribution and a numerical simulation of the speckle amplitude, both after re-scaling (4). Because paper is not an ideal diffuser, the simulation is made, assuming a 2% surface reflection.

Both experiments and simulations give a 10% probability of $C < 1$, a condition preventing the correct operation of the injection interferometer (sign-indicating transitions are lost).

Furthermore, given a fixed spot position, the speckle amplitude changes with the target distance. In particular, we observe strong amplitude variations while performing a measurement along the z axis on a displacement larger than the mean longitudinal speckle size s_z. For such a measurement, the probability to obtain $C < 1$ becomes very high.

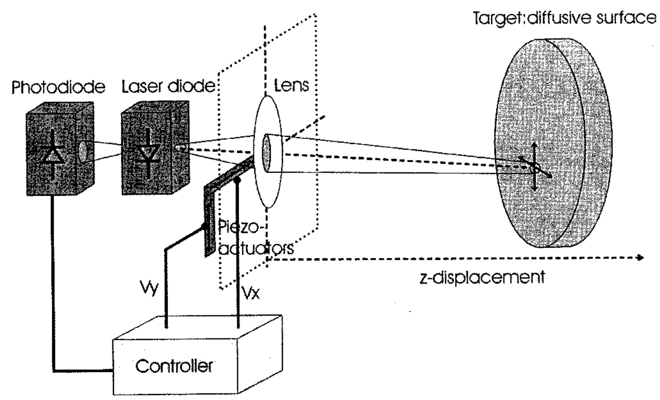


Fig. 4. Experimental setup for self-mixing interferometry with diffusive target.

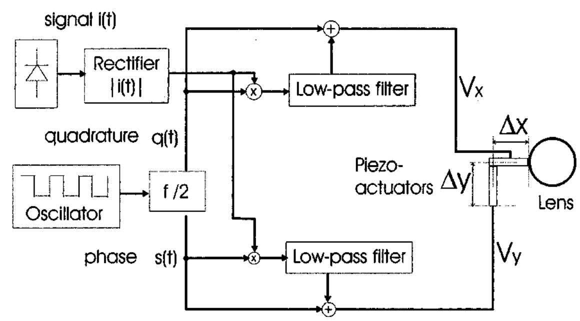


Fig. 5. Block scheme of the controller.

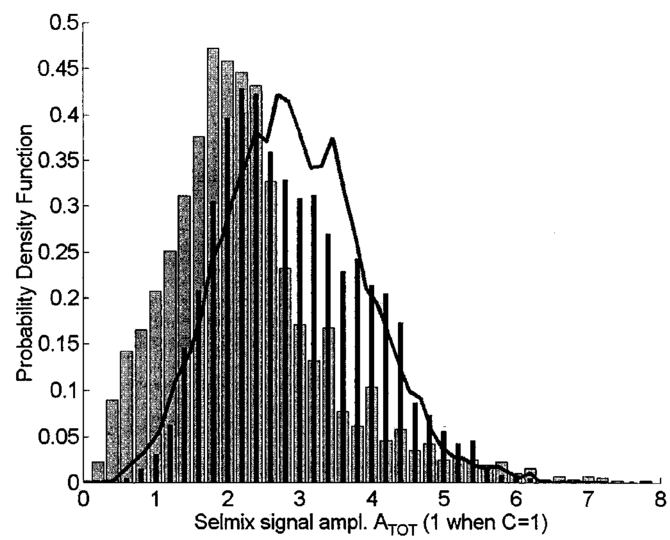


Fig. 6. PDF of the self-mixing signal amplitude when a spot is focussed on a target vibrating 50-cm away from the laser. Grey bars are experimental data with the speckle-tracking system off. Black bars are experimental data with the speckle-tracking system on. Thick lines are numerical simulation of the tracking on the speckle-maximum (surface with 2% reflection and 98% diffusion).

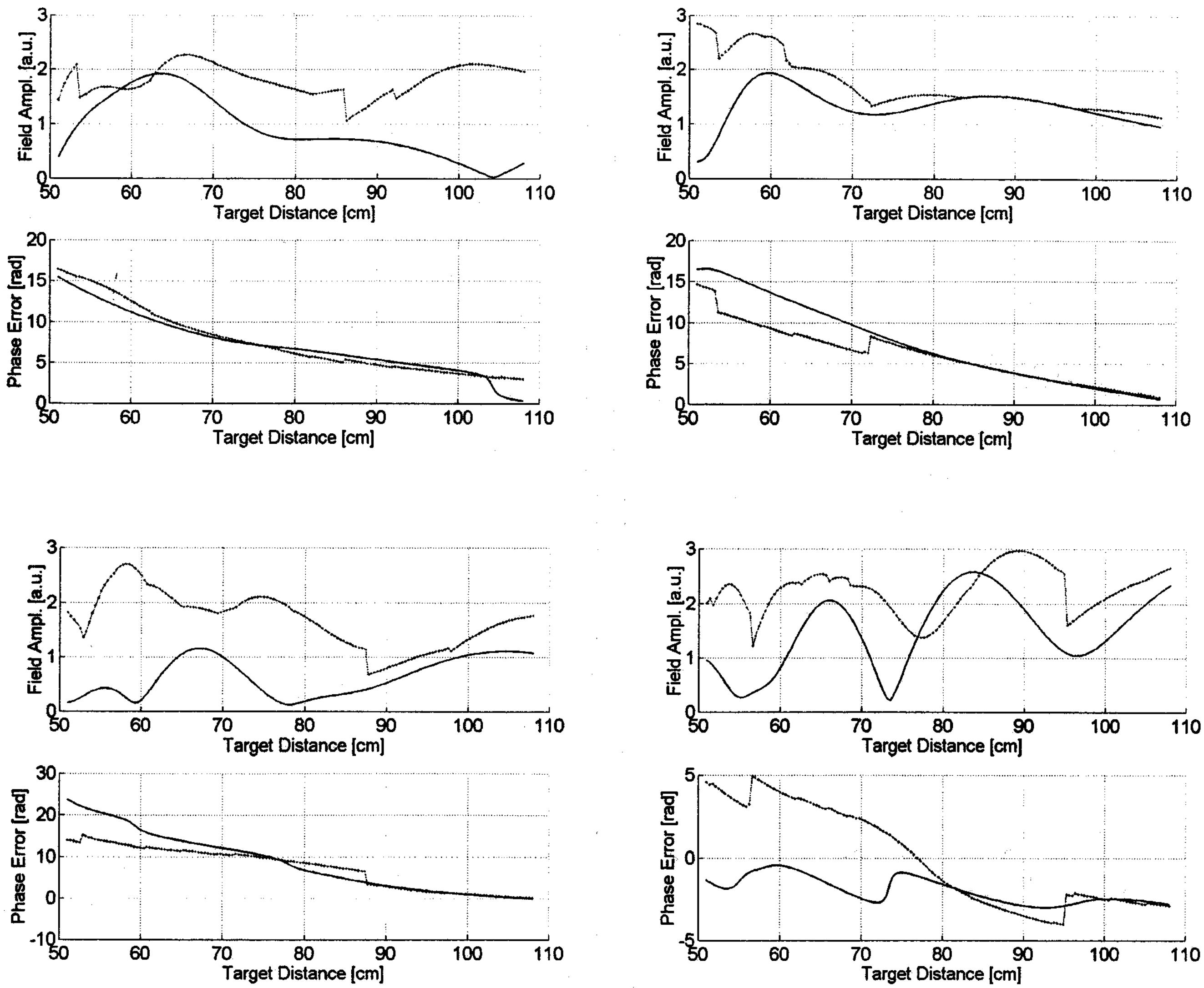


Fig. 7. Examples of simulations of amplitude (top curves) and phase (bottom curves) of the back-injected field. Target moves from 108 to 51 cm. Solid line: field back-injected from a fixed spot, with the peak-tracking off. Light dotted-line: field back-injected with the peak-tracking on.

IV. Speckle Tracking System

A. Working Principle

As the power back-injected into the laser cavity by a diffusive target is strongly dependent on the spot position, we may think of improving the self-mixing signal amplitude by slightly adjusting the spot on the target when amplitude is low, so as to track the peak of a bright speckle. As a method to move the spot, we control the deflection angle of the laser beam, (see Fig. 4), by means of a pair of piezo-actuators moving the focusing lens in front of the laser facet.

The piezo-actuators are driven by two square waves at the same frequency, with a 90° phase shift. This produces a dither of the spot position along a square path, whose size on the target is set to be a few μm (i.e., much less then the spot size).

A control circuit rectifies the self-mixing signal and then multiplies it with the two square waves. After low-pass filtering, we obtain two dc voltages proportional to the signal component in-phase to the square-wave dithering. Adding these voltages to the driving waveform of the piezo-actuators we get, for both axes, a beam movement in the direction of the increasing self-mixing signal. Fig. 5 shows the block scheme of the controller.

B. Speckle Statistics with the Peak-Tracking System

The same acquisition shown in Fig. 3 was repeated with the deflection angle-control on, obtaining the result shown in Fig. 6. It deviates from the Rayleigh distribution especially for low C: the probability of $C < 1$ is now about 0.5% instead of 10% with the control off. Furthermore, we have no evidence of signal fading (i.e., very low values). To evaluate the performance of the tracking method, a simulation of the beam position control has been carried out by calculating the maximum field amplitude that is obtained by moving the spot position in the direction of increasing signal. By repeating the calculation at randomly-selected positions on the target surface, we get the result of Fig. 6. Good agreement with experimental measurements is also found.

V. Displacement Measurement

A. Calculations

In order to evaluate the feasibility of a displacement measurement on diffusing targets, we have calculated the amplitude and phase of the field back-reflected to the laser cavity, as a function of the target distance. The calculation has been repeated for several samples, so as to build the statistics. The diffusing target is simulated by a surface subdivided in individual squares with 1-μm side, each with a random height sorted from a uniform distribution on 0–2λ. The surface was 5 mm on a side and the illuminating laser spot had a radius of 1 mm, to allow for spot position shifts. The results give us an estimate of signal-fading occurrence as well as of phase error due to the speckle-regime.

The simulation of the speckle-peak tracking system is made as follows: at each calculation step, corresponding to a target distance, the spot is shifted along the surface, following the gradient of the back-diffused field-amplitude, until the relative amplitude maximum is reached. Each calculation was repeated with and without the algorithm of speckle-maximum tracking, so as to evaluate the improvement to be expected in the measurement.

A few samples, illustrative of the much larger sample obtained with numerical simulations, are reported in Fig. 7. For each case, we plot the distance-dependence of the field amplitude (top curves) and phase (bottom curves), with and without the peak-tracking. As a check, the average speckle longitudinal size is found to be in good agreement with the theoretical formula, $s_z = \lambda(z/w)^2$, with a spot diameter w of about 2 mm for our collimated beam.

From the diagrams, it is evident that the peak-tracking improves the average signal amplitude and avoids the signal falling to very low values. The abrupt changes, that can be noticed in a few amplitude and phase traces, are the jumps of the algorithm, from a weak speckle to the next bright speckle.

On the 1000-samples of computed displacements (as shown in Fig. 7), the mean phase error is 9.7 rad and its standard deviation is 3.9 rad. The mean error can be traced back to the field curvature φ_{fc}, as can be easily estimated given the spot size. It amounts to a deterministic error of about 1.5 counts ($\sim$0.6 μm), and a speckle error of about $\pm$0.5 counts ($\sim$0.2 μm). The same simulation, made with peak-tracking, gives better results: the mean error is 8.16 rad and standard deviation is 3.6 rad. But, it should be noted that the amplitude fading is greatly reduced.

This simulation run has been repeated for several displacement samples, and the algorithm has statistically given a mean phase error less than about $\pi/2$. This can be qualitatively explained by recalling [3] that, inside a speckle field, amplitude and phase are well correlated. Thus, by keeping the speckle amplitude at a maximum, we also reduce the phase variation, until we jump to another speckle. Going from 50 to 100 cm, we expect a speckle phase error up to a few counts (about 10^{-6} relative accuracy), a value that can be tolerated in many displacement applications.

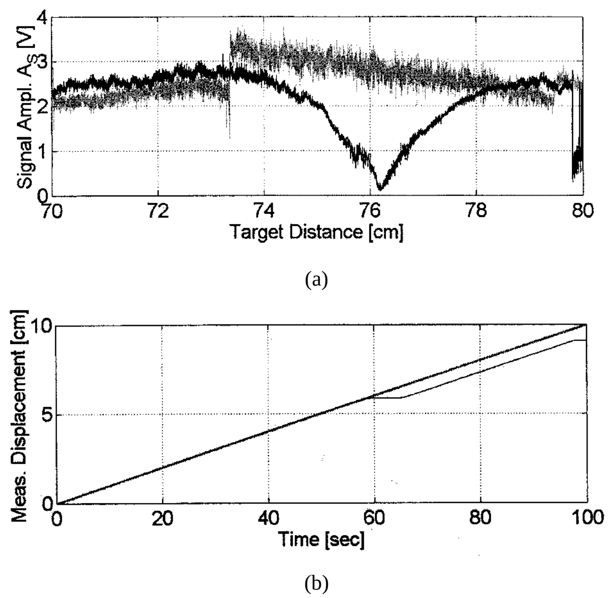


Fig. 8. (a) Comparison between the signal amplitude with (thin black line) and without (thick gray line) the speckle-tracking system. (b) Corresponding displacement as measured by the interferometer. The target was moved from 70 to 80 cm, at a speed of 1 mm/s.

B. Experimental Results

To perform displacement measurements with the spot position control, we have used a linear electrical motor. Positioning of the motor is controlled in a closed loop by an optical ruler having a nominal 1-μm precision. A white paper label, glued in front of the engine, was used as the diffusive target.

Fig. 8 shows examples of the self-mixing signal amplitude, acquired while the target was motor-driven from 70 to 80 cm at a 1 mm/s speed. Without control, the signal amplitude has a strong fading at $z \cong$76 cm. The same measurement, repeated with the speckle-tracking control has no fading, and a speckle jump at $z \cong$73 cm is apparent. The signal looks noisier because of the dither of the spot position given by the piezo-actuators, but this does not affect the correct operation of the interferometer.

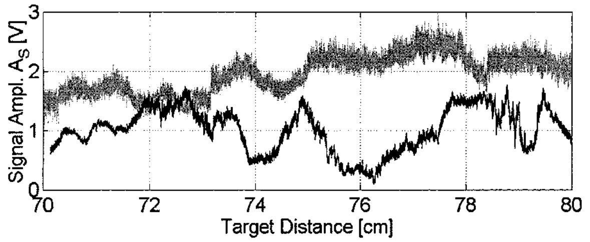
Also noticeable in Fig. 8 is the loss of counts near $z = 76$ cm because of the fading, an error which is eliminated by the speckle-tracking control.

In the measurements, care was taken to avoid lateral shifts of the spot on the paper target during the displacement operation. This would change the speckle sample and introduce an extra error as shown in Fig. 9. Even in this case, however, the speckle-tracking greatly helps to compensate the transversal drift error.

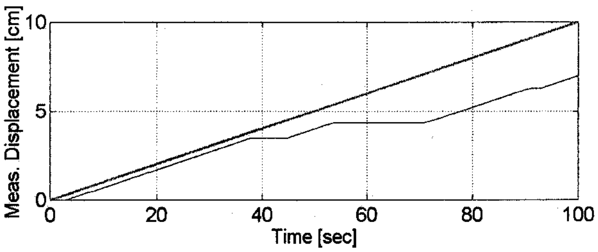
As a last result, we tried to experimentally verify the simulations discussed in Section V-A. Over the distance range 110–60 cm and with the same parameters used to compute the results of Fig. 7, we plot in Fig. 10 the signal amplitude dependence on distance, with the speckle-tracking system on and off. Though bare statistical samples, the general trend of the curves in Figs. 7 and 10 is quite similar.

VI. Conclusion

We have demonstrated, both theoretically and experimentally, the concept of a speckle-tracking system that allows an interferometer to be used on a diffusive target with performance comparable to that for a standard, retro-reflective interferometer.



(a)



(b)

Fig. 9. (a) Comparison between the signal amplitude with (thin black line) and without (thick gray line) the speckle tracking system. (b) Corresponding displacement as measured by the interferometer. The target was moved from 70 to 80 cm, at a speed of 1 mm/s with a slight tilt resulting in a 1-mm lateral displacement.

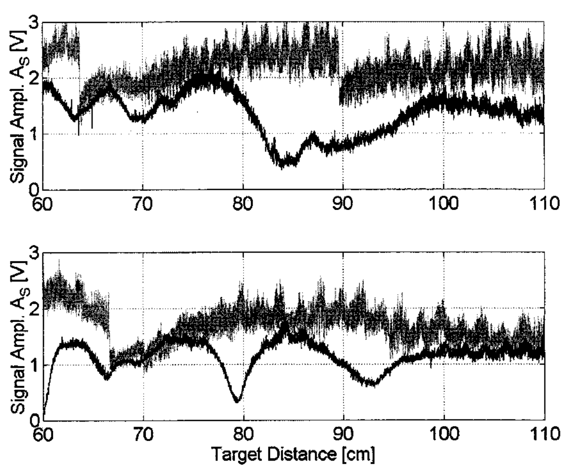


Fig. 10. Two examples of comparison between the signal amplitude with (thin black line) and without (thick gray line) the speckle-tracking system. Paper target moves from 110 to 60 cm at 1 cm/s.

Both amplitude fading and speckle-phase error problems have been eliminated or greatly reduced by the tracking system, which locks the signal to the relative maximum amplitude of the speckle, and has been realized by actuating the laser beam deflection. In addition, we used a self-mixing scheme of interferometry, one requiring a minimum signal-level to work properly. The performance of this system has been validated through a set of simulations, and experimentally by measuring displacements up to 50 cm, on a paper target, with an accuracy of a few parts in 10^6.

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