

Self-Mixing Techniques for Sensing Applications

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Abstract - We present a review of laser diode self-mixing interferometry applied to the measurement of a number of measurands, that we can classify as mechanical (displacement, vibration, distance, angle) as well as physical (remote optical echoes, optical attenuation, coherence length and enhancement factor). After a brief summary of the operating principle, we discuss the applications and report the experimental results about the performance of the sensors.

1 INTRODUCTION

Laser interferometry is one of the most powerful and pervasive techniques in science and engineering, and provides an unparalleled sensitivity to measurement of optical path lengths and derived quantities [1].

Most commonly, laser interferometers are developed from the *external* configuration [1], in which the laser is the source and splitting/recombination of the beams is made external to the source, in an optical interferometer. This is not the only possibility, however, as we can also have an *internal* configuration [1], like in the ring-laser gyroscope, where the laser cavity is also used to develop the optical path length of interest.

In recent years, a third configuration has attracted considerable interest, one in which a fraction of the light back-reflected or back-scattered by a remote target is allowed to re-enter into the laser cavity. This interaction generates a modulation of both the amplitude and the frequency of the lasing field [1].

Variously called *self-mixing*, *feedback* or *induced-modulation* interferometer, the approach uses the laser source as a sensitive detector of the path length $2ks$ (where $k = 2\pi/\lambda$, and s is target distance) travelled by the light to target and back [1]. This is also exploiting the so-called injection-detection scheme [2].

While the first demonstration of this principle were able to detect the Doppler shift of a moving remote reflector [3], the turning point experiments soon followed, developing a complete self-mixing interferometer/vibrometer with a He-Ne laser [4]. In the subsequent years, self-mixing interferometry was widened using a laser diode (LD) as the source/detector [7].

Clear advantages of this new sensing scheme were soon recognised as:

- no optical interferometer external to the source is needed, resulting in a very simple, part-count-saving and compact set-up;
- no external photodetector is required, because the signal is provided by the monitor photodiode contained into the LD package;

- no alignment is necessary, as the laser itself filters out spatially the spatial mode that interacts with the cavity mode;
- sensitivity of the scheme is very high, being quite a coherent detection [2] that easily attains the quantum detection regime (and accordingly, sub-nm sensitivity);
- operation on rough diffusive surface is possible;
- information is carried by the laser beam and it can be picked up everywhere, also at the remote target location [1].

2 PRINCIPLE OF OPERATION

As optical feedback in LDs is a long-time studied topic, both theoretically and experimentally, we will summarise just the basic results here. Readers interested to a full account of the underlying theory may consult Refs. [1,7-14].

A conventional self-mixing configuration is shown in Fig.1a. It is equivalent to a three-mirror cavity, where $P_r = P_0/A$ is the power back diffused or back-reflected by the remote target, being $A > 1$ the power attenuation of the external cavity. Fig. 1b illustrates a simple interpretation for injection-detection [1]: the small reflected field phasor E_r enters the laser cavity and it adds to the lasing field phasor E_0 . The phase of E_r is $\phi(t) = 2ks(t)$, where $k = 2\pi/\lambda$ and $s(t)$ is the distance of the remote target. Hence, the lasing field amplitude and frequency are modulated by the term $\phi = 2ks$. Namely, the AM term is $\cos(2ks)$, and the FM term is $\sin(2ks)$. This detection scheme very closely resembles the well-known optical homodyne [2]. From the phase-and-quadrature signals the path length $2ks$ can be retrieved without ambiguity, and a measurement of the target displacement is possible [1,4].

The simple treatment illustrated above well applies to HeNe and gas lasers [4], but for the case of a single-mode Fabry-Perot LDs some changes are in order. First, the frequency modulation term is not easily retrieved in LDs, and additionally, the large line width of the LD (in the 1-30 MHz range) makes it almost impossible to detect a useful interferometric signal. Second, the intrinsic non-linear nature of the semiconductor active medium, that couples both optical gain and refractive index to the injected carrier density, makes the amplitude modulation term different from the cosine function.

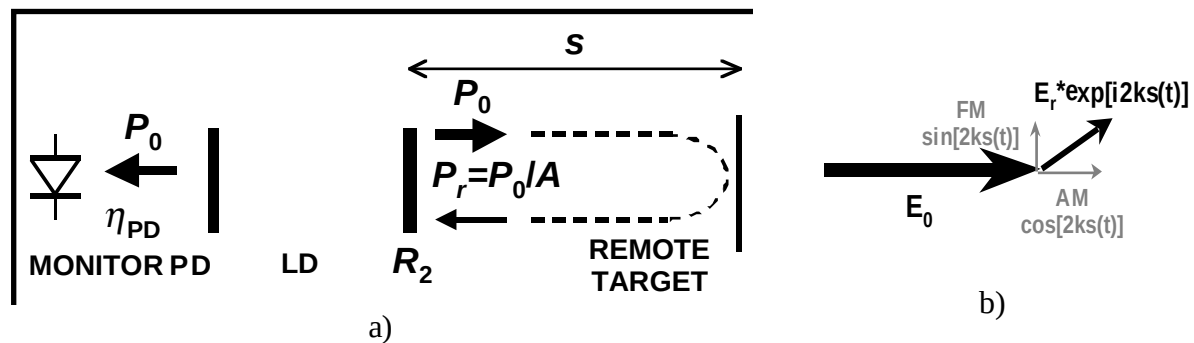


Figure 1. a) The basic self-mixing configuration. b) Rotating vector sum in self-mixing.

A complete analysis of the LD with optical feedback can be performed by using the equations first derived by Lang and Kobayashi [11].

To summarize the effects of the optical delayed feedback, we observe that the back-reflected light interferes with the light already present in the cavity. Depending on the phase of the back-reflected light, the LD threshold condition is varied, and the emitted power changes, as the pump current is held constant. A change in the threshold implies that the actual LD carrier density changes and accordingly, also the emission wavelength is slightly varied. All these self-mixing effect involves a time scale of variation comparable to the carrier LD lifetime, i.e. in the sub-ns range.

An analytical steady-state solution, which is of interest for sensing applications, can be easily found [7-13], leading to the following expressions for the power emitted by the LD:

$$P(\phi) = P_0[1 + m \cdot F(\phi)] \quad (1)$$

where P_0 is the power emitted by the unperturbed LD, m is the modulation index and $F(\phi)$ is a periodic function of the pathlength $\phi = 2ks$, of period 2π . The modulation index m and the shape of the function $F(\phi)$ depend on the so-called *feedback parameter* C [7,8]:

$$C = \frac{\kappa s \sqrt{1 + \alpha^2}}{L_{\text{las}} n_{\text{las}}} \quad (2)$$

where α is the linewidth enhancement factor, L_{las} is laser cavity length, n_{las} is cavity refractive index and κ is given by: $\epsilon (1-R_2)/\sqrt{A}\sqrt{R_2}$, where $\epsilon \leq 1$ accounts for a mismatch between the reflected and the lasing modes, A is total power attenuation in the external cavity and R_2 is LD output facet power reflectivity (see Fig.1a). Thus, the value of the C parameter depends on both the amount of feedback and, interestingly, on target distance s . The C parameter is of great importance, as it discriminates between different feedback regimes:

- $C \ll 1$ is for the *very weak feedback* regime. Here, function $F(\phi)$ is a cosine (like in gas lasers), and modulation index m is proportional to $A^{-1/2}$.
- $0.1 < C < 1$ is the *weak feedback* regime. Function $F(\phi)$ gets distorted and has a non-symmetrical shape (Fig.2a); modulation index is still $\propto A^{-1/2}$.
- $1 < C < 4.6$ is the *moderate feedback* regime. Function $F(\phi)$ becomes three-valued, i.e. the system becomes bistable, with two stable states and one unstable (Fig.2b); modulation index m still increases for decreasing A , but it is no longer proportional to $A^{-1/2}$. The interferometric signal is sawtooth-like and exhibit shysteresis.
- $C > 4.6$, is the *strong feedback* regime. Function $F(\phi)$ becomes five-valued or

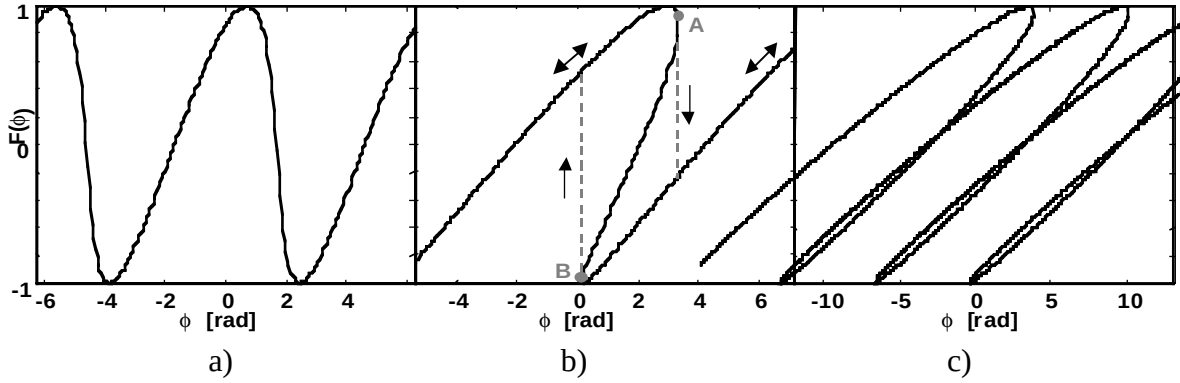


Figure 2. Calculated waveforms of the function $F(\phi)$ vs. ϕ for different values of the feedback parameter C . a) $C = 0.7$; b) $C = 3$; c) $C = 10$. In b), the segment AB is unstable. When the system is in A and ϕ further increases, it jumps down along the dashed line.

When the system is in B and ϕ decreases further, it jumps up.

even more (Fig.2c), and not all the specimen of F-P LDs remain in the self-mixing regime; rather, in some cases the LD enters the mode-hopping region and interferometric measurements are no longer possible.

The calculated waveforms of function $F(\phi)$ for three representative feedback regimes are shown in Fig.2a-c. In Fig.3 we report the experimentally measured self-mixing signals, obtained when the phase ϕ of the back-reflected field is sine-wave varied by vibrating the remote target with a loudspeaker.

The resulting self-mixing signal is a periodic function of ϕ , and a complete interferometric period appears each time phase is varied by 2π . Consequently, the fringe period corresponds to $\lambda/2$ displacement. Some remarks are in order:

- ❖ The asymmetry in the shape of function $F(\phi)$ allows discriminating the direction of motion of the target [7,14]. This is a crucial point, and taking advantage of it we make a non-ambiguous interferometer displacement measurement using just *a single interferometer channel*.
- ❖ In the moderate feedback regime, the modulation coefficient m is in the range 0.5-5 %, small but adequate for subsequent signal processing.
- ❖ The self-mixing signal is readily obtained in any type of single-longitudinal mode F-P LD with good side-mode suppression. We found good LDs with λ ranging from visible (635 nm) to the 3rd fiber-window (1550 nm).

3 APPLICATIONS TO METROLOGY

3.1 Displacement Measurements

To detect the displacement of a target, the LD is driven by a constant current, and a lens is used to focus or collimate light onto the target. Different from other approaches, using a self-mixing interferometer, the target can be reflective (i.e., mirror), retroreflective (i.e., a corner-cube or the 3M Scotchlite™ reflective paper) or diffusive (i.e., a rough surface).

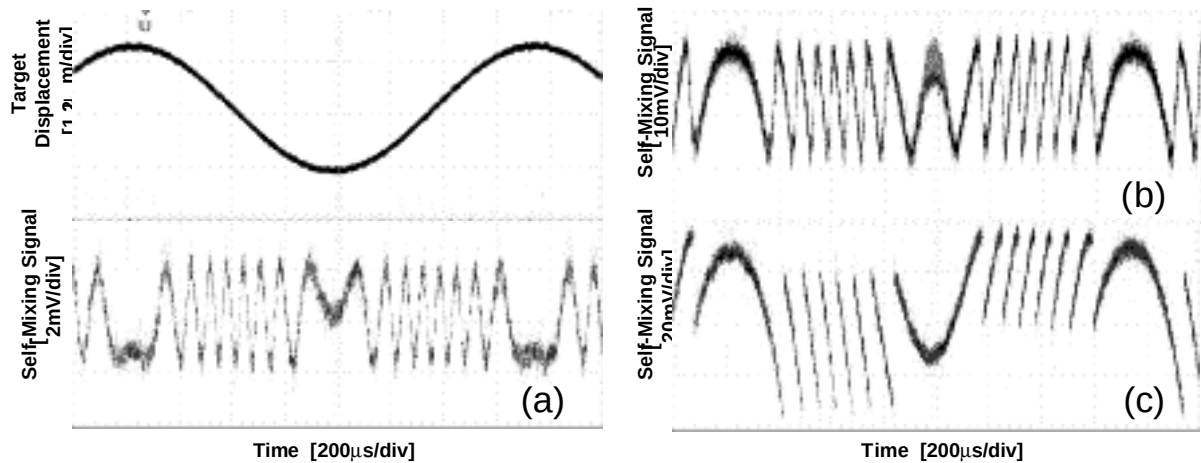


Fig. 3 Self-mixing signal waveforms obtained experimentally for different values of the total optical attenuation A . Upper-left trace: loudspeaker drive signal at 657 Hz, 1.2 $\mu\text{m}/\text{div}$; a) $A \approx 2 \cdot 10^8$, $C \ll 1$; b) $A \approx 8 \cdot 10^6$, $C \approx 1$; c) $A \approx 4 \cdot 10^5$, $C > 1$.

The only caution with target type, is that an optical attenuator may be required, to be inserted along to light path to avoid excessive back-reflection.

The easiest way to build a displacement sensor is to operate the LD in the moderate feedback regime ($C > 1$). So, the self-mixing signal is saw-tooth and the sign of the fast transitions depends on the target direction of motion. Target displacement can be retrieved with $\lambda/2$ resolution (i.e. ≈ 325 nm with a visible LD) without sign ambiguity. To do so, we perform an analogue derivative of the self-mixing signal and count the occurrence of negative and positive pulses, as shown in Fig.4a [7]. An experimental self-mixing signal for a vibrating target is reported in Fig.4b, showing the fast upward and downward pulses. Using this approach with a retro-reflective target, displacement has been successfully measured over 1 m distance, with an allowed maximum speeds of 0.4 m/s, solely limited by electronics bandwidth.

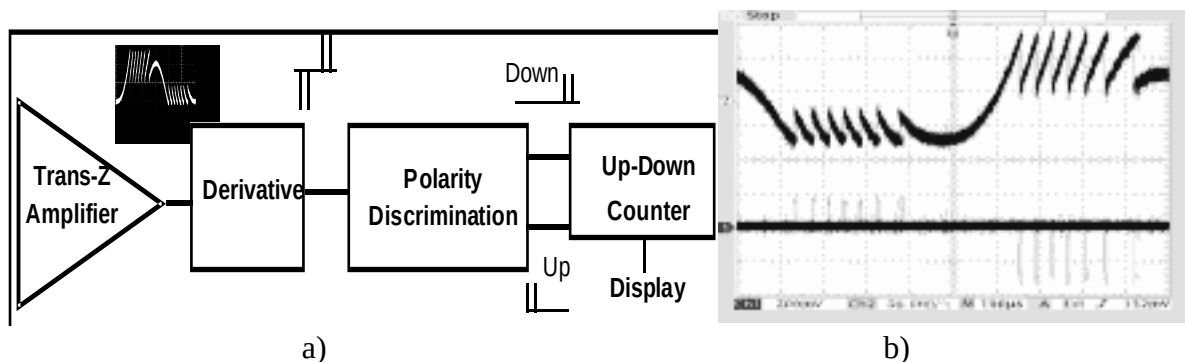


Fig.4: a) Electronic signal processing schematic for a fringe-counting displacement interferometer. b) Upper trace: experimental self-mixing signal obtained for a sinusoidal target displacement of 3.3 μm (peak-to-peak) amplitude and 1 kHz frequency; lower trace: analog derivative of self-mixing signal, showing up/down pulses. Time scale: 100 $\mu\text{s}/\text{div}$.

The maximum target distance is obviously limited by LD coherence length: it can reach 7-8 m when using moderate power 780 nm LDs intended for CD pick-ups. Starting from the basic set-up, improvements can be made in two directions, i.e., increasing the resolution and allowing operation on diffusive surfaces.

Resolution can be improved through two different approaches:

- i) Fast modulation of the interferometric phase through an LD current modulation, that causes an LD wavelength shift $\Delta\lambda$. The wavelength shift produces a phase dithering, i.e. $\Delta\phi = 2 \cdot 2\pi / \lambda_0^2 \cdot s \cdot \Delta\lambda$, where s is LD-to-target distance. By properly sampling the self-mixing interferometer signal synchronously with the dither, resolution can be increased.
- ii) Sampling the self-mixing signal, and processing it with the aim of inverting the function $F(\phi)$, so that the target displacement is exactly reconstructed, as shown in [14]. In this case, we need to know parameter C , or make a pre-processing of the self-mixing waveform to determine its actual value. A simplified version is to linearize the self-mixing waveform (i.e., function $F(\phi)$ is approximated by a sawtooth).

By using one of the two above methods, resolution improvements of a factor 10 have been demonstrated, i.e. ≈ 40 -nm accuracy has been achieved [14,15].

Operation of conventional displacement measuring interferometers requires a co-operative target and an accurate alignment procedure. Typically, the target is a corner-cube mounted on the moving object under test, a requisite accepted by the users' community, reluctantly. But, it would be much better being able to work directly on a diffuser surface as found in the normal workshop environment, with no invasiveness nor the need to keep optical surfaces clean.

This chance is actually offered by the self-mixing configuration because it is intrinsically self-aligned and it is effective even for the case of very small optical back-reflections.

However, with diffusive targets practical limitations of operation can occur due to speckle-pattern effects [16,17], especially for the case of target displacements larger than a few mm, because the speckle distribution may change randomly, thus causing signal fading.

This problem is obviously common also to conventional interferometer techniques, which in turn are faulty and not reliable for these applications. However, the self-mixing approach in conjunction with an appropriate "bright" speckle tracking system allowed to solve this problem, and was demonstrated as the first interferometer capable of working satisfactorily even on a diffusing surface [16,17].

The method employed to avoid amplitude signal fading is based on a slight change imparted to the laser spot position on the target in the transversal direction [16].

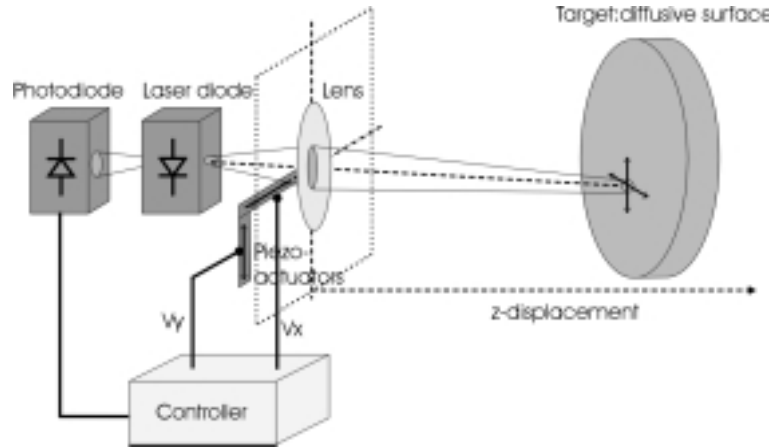


Fig.5. Experimental optical head arrangement for speckle-tracking in a self-mixing interferometer (DL is HL7851G, 50 mW @ 780nm), including the two piezo actuators and a controller with phase detection to dynamically lock the spot on a bright speckle.

The spot movement is obtained by means of a pair of piezo-actuators holding the focusing lens that controls the deflection angle of the laser beam.

The experimental arrangement is shown in Fig.5.

Attempting to make an interferometer measurement on a diffusing surface, reveals problems of amplitude fading and phase errors.

First, the signal power returning to the beam-splitter from a diffuser is reduced of a factor $N=A\Omega/\lambda^2$ respect to the mirror target, where N is the number of spatial modes of the target, as given by the target area A and the radiating solid angle Ω ($\Omega=\pi$ for an ideal diffuser) [1,2]. However, the signal reduction only impacts the minimum-detectable-displacement, as even with a very small signal power, the interferometer performs a coherent detection. Thus, the quantum limit of S/N is always attained, and the minimum-detectable-displacement is $MDD= \lambda/2\pi(S/N)$ [1], with practical values (nm to pm) still satisfactory even for high (10^4 - 10^6) attenuations.

Second and more of concern, the average power $\langle P \rangle$ is subjected to the speckle-regime statistics, and has a probability density [1,2,16, 17]

$$p(P) = \frac{1}{\langle P \rangle} e^{-P/\langle P \rangle} \quad (3)$$

Small amplitude speckles are relatively frequent (e.g., 10% have less than 10% the average power), thus signal can be lost by fading when moving longitudinally the target along speckles.

Thus, the $C>1$ condition of strong-injection is harder to match. We can evaluate the probability of $C>1$ by studying the speckle distribution. By acquiring the amplitude of the self-mixing signal we can calculate the intensity of the back-injected power: in Fig.6 we plot the statistics measured for a paper-surface target (grey bar), which is in good agreement with the exponential distribution.

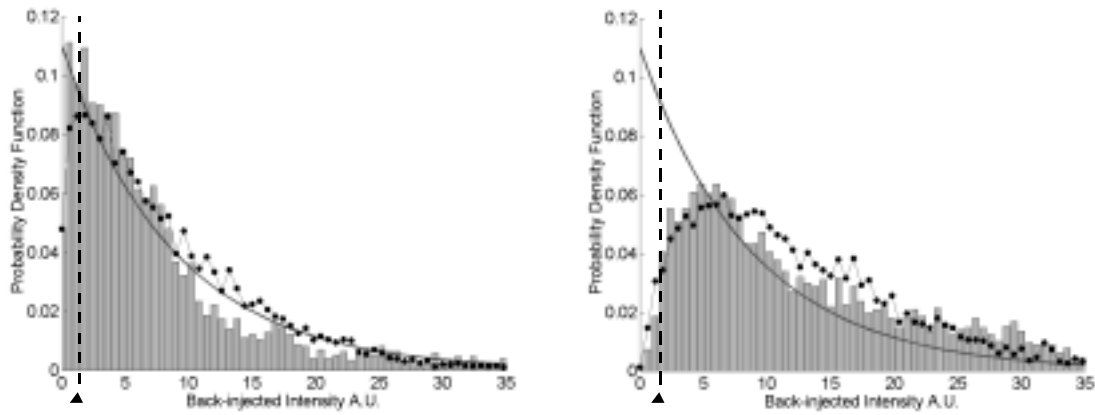


Fig. 6 Experimentally measured probability (pdf) of backscattered light intensity from a paper target placed at 0.5-m from the LD. Statistics is obtained by sampling different transversal positions of the target. Horizontal axis is normalized to a feedback parameter $C = 1$. Hence, vertical dashed line sets the threshold for correct displacement measurement by fringe counting. Gray bars are experimental data. Thick lines are theoretical exponential pdf's. Dotted lines are data from a simulation [16]. Left: speckle-tracking system turned off; right: speckle-tracking system turned on.

As the power back-injected into the laser cavity by a diffusive target is strongly dependent on the spot position, we improve the self-mixing signal amplitude by moving away the spot on the target when amplitude is low, so as to track the peak of a bright speckle.

As said above, we control the deflection angle of the laser beam (Fig.5) by means of a pair of piezo-actuators moving the focusing lens placed in front of the laser facet.

The piezo-actuators are driven by two square waves at the same frequency, with a 90° -phase shift. This produces a dither of the spot position along a square path, whose size on the target is set to be a few μm (i.e., much less than the spot size and practically unnoticeable). A control circuit rectifies the self-mixing signal and multiplies it with the two square waves. After a low-pass filtering, we obtain two dc voltages proportional to the signal component in-phase to the square-wave dithering [17].

Adding these voltages to the driving waveform of the piezo-actuators we move the beam, for both axis, in the direction of the increasing self-mixing signal.

In Fig.7 we compare the intensity statistics with and without the speckle tracking system, as we move substantially the target, from 70 to 80 cm from the laser. The figure is a typical sample of the statistics we have observed upon repeating the experiment several times.

In the figure, when a dark speckle is found, a measurement error is introduced with the tracking system off, whereas the error is removed switching the tracking system on.

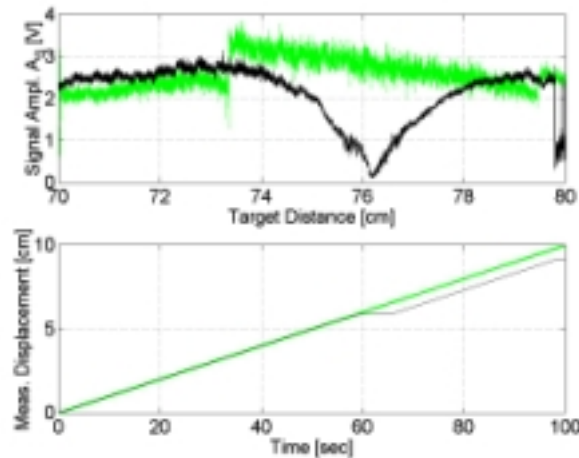


Fig. 7: Top: signal amplitude with (thin black line) and without (thick gray line) speckle-tracking system. Bottom: the corresponding displacement as measured by the interferometer.

3.2 Vibration Measurements

Laser vibrometry is a remote sensing technique capable of measuring zero-mean displacement of a (generally rough) surface under test [18,19]. The self-mixing scheme proved to be efficient also for this application. The principle, shown in Fig.8, relies on operation in the moderate feedback regime (i.e., triangular waveform) by locking the interferometer phase to half-fringe [18]. Hence, by means of a suitable feedback loop acting on LD wavelength, environmental low-frequency phase fluctuations can be cancelled out, and a vibration of amplitude smaller than $\lambda/4$ can be linearly translated into an electrical signal. The ultimate sensitivity is then set by the quantum noise associated with the detected signal, which can be expressed [2] in terms of NED, noise equivalent displacement, as:

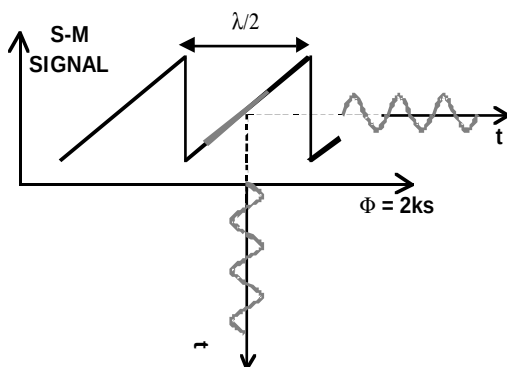


Fig.8. Principle of vibration measurement by locking to half-fringe.



Fig.9. Nomogram reporting the vibrometer performance in the frequency-amplitude plane

$NED = (\lambda/2\pi)/(SNR)$ where SNR is the signal-to-noise ratio of the self-mixing signal. The experimentally obtained sensitivity is 10 pm/ $\sqrt{\text{Hz}}$ [18], a remarkable figure indeed.

Extension of the dynamic range can be achieved by a phase-nulling technique, in which the electronic feedback loop also compensates for interferometer phase changes caused by the target motion. By differentiation of the phase expression, we obtain: $\Delta\phi = 2 \cdot 2\pi/\lambda \cdot \Delta s - 2 \cdot 2\pi/\lambda^2 \cdot s \cdot \Delta\lambda$. Hence, the phase change caused by a small target displacement Δs can be compensated by applying a wavelength variation $\Delta\lambda$ to the LD such that $\Delta\lambda = (\lambda/s)\Delta s$. This is accomplished by the electronic feedback loop, that generates an amplified “error signal” voltage which is proportional to the vibration Δs .

Based on the above exposed principle, an engineered prototype instrument has been designed, built and its performance tested [18]. The instrument consists of two units. The optical head includes the 800 nm single-mode F-P LD, a collimating lens, a focussing output lens and the transimpedance amplifier. It is connected to an electronic unit that contains power supply, electronic feedback loop, LD current source, and processing electronics needed to display the RMS amplitude and the frequency of the measured vibration. With the optical arrangement chosen for the vibrometer optical head, the maximum measurable vibration amplitude is $\Delta s_{\max} = 150\text{-}\mu\text{m}$ peak-to-peak. Beyond this vibration amplitude, locking to the interferometric fringe is lost, the system jumps onto the next interferometric fringe, and the output signal gets distorted. To further extend the instrument dynamics, an circuit compensates the distortion appearing on the output signal, and the maximum measurable vibration $\Delta s_{\max}$ reaches 600 μm peak-to-peak, solely limited by the speed of the electronics.

Instrument performance is summarised by the nomogram of Fig.9. A sensitivity of 100 pm/ $\sqrt{\text{Hz}}$ has been obtained, on a 100-Hz to 70-kHz frequency range. At lower frequency, sensitivity is not intrinsically degraded, but 1/f mechanical vibration noise from environment becomes relevant. Maximum measurable vibration of 600- μm peak-to-peak is obtained for frequencies 2 to 200 Hz. At higher frequencies, the maximum measurable vibration is limited by electronic circuits speed, and in principle it can be further increased. The small-signal high-frequency cut-off is 70 kHz.

3.3 Absolute Distance Measurements

Measuring the absolute distance to a remote target by means of an incremental method has been the dream of measurement scientists since the advent of the laser. Indeed, as interferometric and phase measurements are incremental, there appeared no hope left of carrying out a measurement which is non-incremental (absolute) on a large number of wavelengths.

But, taking advantage of the λ -modulation we can impress by the LD drive current, researchers have been able to circumvent the requirement of moving

the target and developing the counts to be accumulated [6,20-22]. Rather, we count pulses developed by the self-mixing signal at $C>1$ for each full period the path length contains an exact number of wavelengths [20,21].

From an analysis of the phenomenon [1,20], we find that distance is measured with a resolution given by:

$$\Delta s = \lambda_0^2 / 2\Delta\lambda \quad (4)$$

where λ_0 is the center wavelength of the DL and $\Delta\lambda$ is the wavelength swing produced by a drive current modulation ΔI .

The quantity Δs in Eq.4 is in practice limited to about 0.3 to 1-mm, because the swing $\Delta\lambda$ cannot be too large without incurring in mode-hopping with associate error. In a companion paper to this conference [22] we present a method to improve resolution down to probably 10- μ m, on a range of $s=0.2$ to 3-m typically.

3.4 Angle Measurements

Alignment of mirrors along the line of sight defined by a laser-beam wave vector is one of the early applications reported for injection interferometry [23]. In a straight arrangement for autocollimator measurement, the laser beam is expanded by a telescope and sent to the mirror target [1,24].

On a substantial distance (say, >30 cm) the normal ambient-induced vibrations already provide an interferometric signal, and amplitude is maximum when the alignment is best.

More specifically, the maximum is attained when the angle error α is fairly less than the diffraction limit of the transmitted beam, with a typical resolution <3 arcsec.

The injection-interferometer autocollimator has been found useful in alignment with IR lasers (originally, with a 3.39 μ m He-Ne laser) [23]. However, it does not provide a true angle measurement, but just a sensitive null detection.

By adding a line-of-sight modulation to the basic scheme, we can work out a true angle measurement. The beam can be steered by an actuator, e.g., a x-y translation of the laser chip or of the first lens of the telescope, or by means of a prism inserted in the transmitted beam. To substantiate the design, let us assume a deflection $\Delta x = \Delta x_0 \cos \omega_0 t$ of the first lens. Then, the angle of k is modulated with a deflection $\alpha = \Delta \alpha_0 \cos \omega_0 t$, where $\Delta \alpha_0 = \Delta x_0 / F$ and F is the focal length of the first lens.

As illustrated in Fig.10 (from Ref.[1]) with no modulation the response versus α is parabolic, whereas adding the modulation and using phase detection, the response is linear up to the amplitude $\Delta \alpha_0$ of the angle swing.

Typical performance of the angle meter implemented with a laser diode in the injection-interferometry configuration [24] is a noise-limited resolution of ≈ 0.2 arcsec on a ≈ 5 arcmin dynamic range.

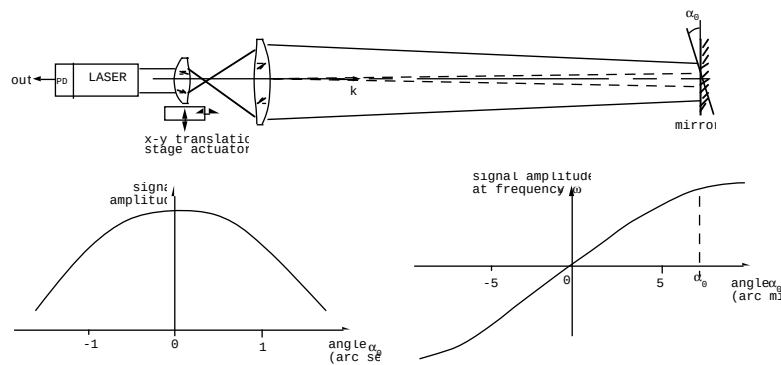


Fig. 10. Angle measurement by self-mixing interferometry: the aiming direction is modulated so that the parabolic amplitude response is transformed into a linear dependence on the angle α_0 to be measured (from [1]).

3.5 Velocimetry

Single beam as well as two-beam (or the so-called Doppler) velocimetry have been demonstrated since the early times of self-mixing interferometry [1,3,5,24-27]. Some of the papers reporting a self-mixing velocimeter were actually on vibrometers (Sect.3.2), i.e., described an instrument providing the v_z component of the target motion (the z-axis being the line-of-sight) and operating on a surface rather than on particles dispersed in a fluid.

To have a true velocimeter (or LDV) measuring the out-of-plane (v_x or v_y) velocity component, we shall start with a laser with a good self-mixing effect and add an appropriate optical section [1]. This section will be composed of the usual beamsplitter, mirror, and objective lens arrangement required for the two beam arrangement of the basic schematic of the Doppler velocimeter [1]. In this configuration, the field returning from the scattering volume is collected by the objective lens and fed into the laser cavity.

Here, the injection generates the usual AM- and FM- induced modulation. If we limit ourselves to detect the easy amplitude modulation component, it will suffice to place a photodetector on the rear mirror of the laser to get the desired LDV signal.

Of course, the self-mixing LDV can only operate in the backward-scatter configuration. Compared to the external interferometer configuration, there is the extra loss of mirror transmittance, but optical part count is the minimum.

4 MEASUREMENTS ON MICRO-ELECTRO-MECHANICAL SYSTEMS (MEMS)

Characterization of Micro-Electro-Mechanical-Systems (MEMS) is important in the evaluation and testing of fabricated microstructures. In particular, self-mixing interferometry is an innovative optical tool for non-contact measurements on MEMS [28-29]. The MEMS under test can be modeled as

mass-springs resonating systems. The device illustrated in Fig.11 consists of a single laminar mass suspended in the horizontal x-y plane by springs, attached to the substrate in a few points. This single-mass, or one-proof-mass, structure, proposed to fabricate dual-axis linear accelerometers or gyroscopes [28], is equipped along the sides with arrays of capacitors, useful for actuation and/or detection.

The principle of operation of this structure is based on the detection of the Coriolis' force F_C [1,28,29]. We apply a harmonic force $F = F_0 \sin \omega t$ to the driving axis, and displacements on both axes are harmonic at the same frequency. If the resonance frequencies are matched and excitation occurs at $\omega = \omega_{res}$, the vibration amplitude on the sensing axis is maximized and is given by $Y_s = Q_y F_C / K$, where Q_y is the quality factor, and $F_C = 2mv\Omega$ is the Coriolis force, v being velocity along x-axis, m the mass and Ω the angular velocity perpendicular to the xy-plane. Thus, the displacement Y_s is proportional to the angular velocity Ω to be detected.

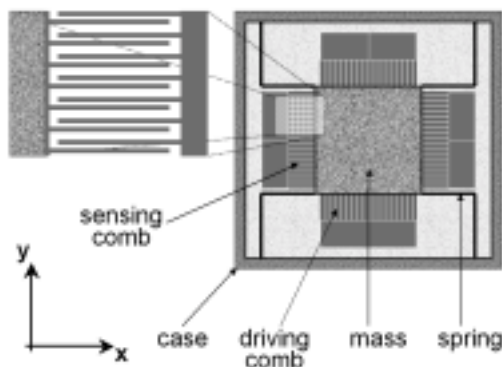


Fig.11. Schematic layout of a single-mass device. In the sketched example, parallel plates structures are employed

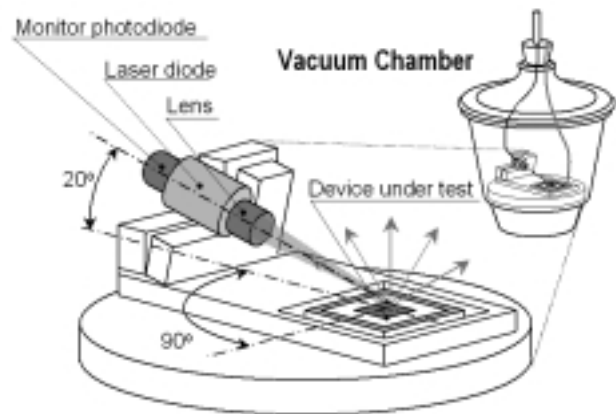


Fig.12. Setup of the self-mixing interferometer for MEMS testing

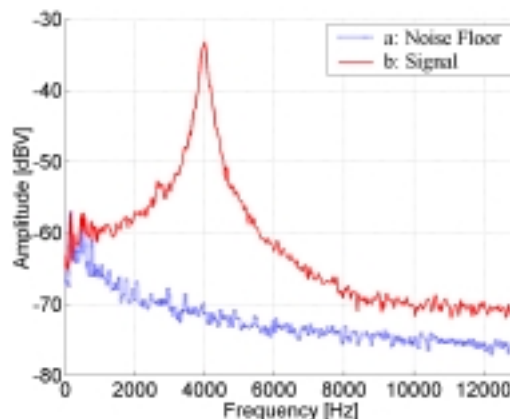


Fig.13. Typical resonance of a MEMS suspended mass, at atmospheric pressure.

The optical head incorporates only a laser (LD), a photodiode (PD), a collimator or focusing lens, and eventually a front-end transresistance amplifier (Fig.12).

In our experimental arrangement, for detecting vibration modes in the horizontal plane x-y, the device under test is positioned at an angle $\alpha=20^\circ$ with respect to the laser beam, with laser/target distances of 6-cm. Selectivity to modes detection may be obtained by changing the relative direction of the laser beam and the displacement. This feature, in principle available with any interferometric configuration, is easily implemented with our very compact setup, which fits into a small vacuum chamber, thus allowing MEMS testing at different pressures.

The amplitude resonance curve can be directly visualized, in real time or with a short averaging, by feeding an electrical spectrum analyzer with the photodetected signal and applying white noise to the device under test, as the time-varying component of the electrical driving signal. In such measurements, linear operation is maintained by operating the interferometer in quadrature and for displacements smaller than $\lambda/4$ [1,29]. The frequency response of a single-mass device at atmospheric pressure is shown in Fig.13.

Still another intriguing possibility offered by self-mixing interferometry is to build a gyroscope as a hybrid optical MEMS (or MOEMS) [30]. Here, we use a conventional mechanical structure and make a very the high-sensitivity readout readout of the small Coriolis-induced displacement by means of the self-mixing interferometer, in form of a chip incorporated in the MEMS case [1,30].

5 MEASUREMENT OF PHYSICAL QUANTITIES

Besides the applications aimed at the measurement of properties and mechanical quantities of remote objects/targets, the self-mixing technique can be useful to gain further insight into a number of physical quantities [1], as we briefly outline in this Section.

5.1 Detection of remote echoes

Besides the phase difference $\phi_m - \phi_r$, the generic interferometric signal contains information on the amplitude of the field E_m returning to the measuring port. Actually, any interferometer can be regarded as a coherent detection scheme [2] in which E_m is the signal and $E_r \gg E_m$ is the local oscillator. By beating of the fields E_r and E_m at the photodetector, an internal gain is generated, and the quantum limit of detection is attained.

The coherent detection scheme is classified as homodyne or heterodyne according to whether $f_m = f_r$ or $f_m \neq f_r$ and as an injection scheme if the returning signal is fed into the source.

We can write the photocurrent signal $i_{ph} = \sigma \Delta P$ of the self-mixing configuration in the weak-injection regime as: $i_{ph} = I_0 \kappa a \cos 2ks$.

Here, $I_0 = \sigma P_0$ is the dc photocurrent, $\kappa = c\tau_p/n_l L$ is a constant of the order of unity, and a is the total (field) loss in the path to the target at distance s and back, explicitly $a = \sqrt{A}$ in terms of the total power attenuation A .

At weak injection levels ($a \ll 1$), the signal has an amplitude proportional to the square root of power attenuation, $a = \sqrt{A}$, as expected from a coherent detection process. When $\kappa a \approx 1$ or $C > 1$ (moderate feedback), the amplitude does not increase any more and we get a saturation of the signal versus attenuation. To be able to measure the amplitude $I_0 \kappa a$ without being disturbed by the phase term $\cos 2ks$, we need either the distance to be a constant or to be varying in a known way. If we are operating on a substantial distance (say $s > 50$ cm), ambient-related microphonics usually contributes with a random jitter $s_j(t) \gg \lambda$ added to the mean $\langle s \rangle$, and therefore $\cos 2ks$ is a random waveform with zero mean value and a rms value $1/\sqrt{2}$.

If distance is short or we want to move the signal off the dc, we may add in the optical path a phase modulation $\Phi = \Phi_0 \cos \omega_m t$, with a deviation $\Phi_0 > 2\pi$ large enough to have a phase term $\cos(2ks + \Phi)$ swinging from -1 to $+1$. In this way, the signal is modulated on a carrier frequency ω_m , at which the measurement of amplitude will be performed.

Typical examples of echo attenuation measurement [1,31-33] are shown in Fig.14. The first setup is for the measurement of the return loss from a fiber device (DUT, device under test). The optical path length is modulated with the aid of a piezoceramic (PZT) phase modulator, driven at frequency ω_m , and the output signal at frequency ω_m is proportional to the square root of the ratio $P_{\text{back}}/P_0 = A_{\text{RL}}$. In this scheme, we may also add a second photodetector PD2 to measure the ratio P_{tr}/P_0 , of the DUT, that is, its insertion loss.

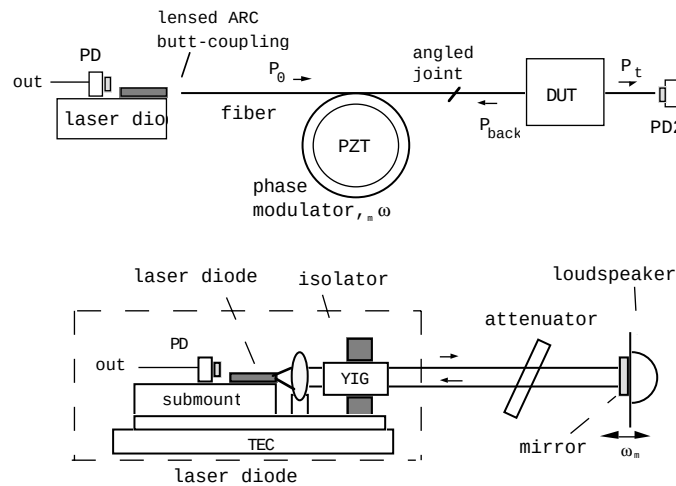


Fig.14. Typical arrangements for the measurement of weak echoes by self-mixing interferometry. Top: for measuring the return loss of an all-fiber device (DUT), we use an in-line PZT phase modulator. Bottom: for testing the isolation of an optical isolator, a remote vibrating mirror on a loudspeaker supplies the path length modulation.

In the second example, aimed to test the isolation factor of an optical isolator mounted in front of the laser chip [32], we shall use a path-length modulation external to the device. This can be a mirror aligned to the transmitted beam and mounted on a loudspeaker, driven at the desired frequency ω_m . In this case, the signal is proportional to the square root of the isolation factor P_{back}/P_0 .

To make the point clear, 10 dB (or a decade) of attenuation corresponds to a half a decade of variation in the current or voltage signal, that is, to a 10 dB change in it.

Typical performances of the echo detector based on the injection interferometer are reported in literature [1,31-34]. Several single-mode laser diodes operating at different wavelengths have been experimented, with either plain Fabry-Perot or DFB (grating reflector) structure. For all of them, the range of measurable attenuation spans from -25 dB to -80 dB. By adding an attenuator in the optical path (Fig.14), we can extend the upper limit to about 0 dB.

In CDs, optical pits correspond to the bits of recorded information. In conventional design, to read the bits we need a laser and photodiode combination, a beamsplitter to divide the input and output beam paths, and a conjugating lens to focus on the spot.

Using an injection detector, we can dispense for the beamsplitter and the lens by placing the laser diode close to the disk (typically at 10-20 μm off the surface). The rear photodiode will supply the readout signal in form of spikes corresponding to the bits superposed to the dc quiescent current [34].

5.2 Laser line-width

The SL line-width can be evaluated from measurements of the phase noise of the self-mixing interferometer performed for different target distances, as the phase noise is proportional to both SL line-width and target distance L . The self-mixing scheme, in the moderate feedback regime (feedback parameter $C > 1$, saw-tooth waveform), offers two advantages. First, due to the fast switching, the phase noise can be evaluated by simple time-domain measurements instead of spectral density analysis in the radio frequency (RF) range. Second, a compact set-up can be used, with overall length much shorter than the SL coherence length, in contrast to the other well-known, non-RF methods [35].

When light from a laser source enters an interferometer, the fluctuation of the laser frequency (represented by the line-width $\Delta\nu$) generates phase noise. For a self-mixing interferometer the RMS phase noise is obtained as:

$$\sqrt{\langle \Delta\phi^2 \rangle} = \frac{4\pi}{c} \sqrt{V_0^2 \langle \Delta L^2 \rangle + L_0^2 \langle \Delta\nu^2 \rangle} \quad (5)$$

where ν_0 is SL mean emission frequency, L_0 the target distance and ΔL its random fluctuation. If phase noise is measured for different values of target distance L_0 so that the first term under square root in (5) is small respect to the second term, then a linear dependence $\sqrt{\langle \Delta \phi^2 \rangle} \propto L_0 \sqrt{\langle \Delta \nu^2 \rangle}$ is obtained, and the slope of the $\sqrt{\langle \Delta \phi^2 \rangle}$ curve vs. L_0 is proportional to the line-width $\sqrt{\langle \Delta \nu^2 \rangle}$. So, SL line-width can be recovered from subsequent measurement of the RMS phase noise performed at different target distances.

The saw-tooth self-mixing signal allows an easy and accurate measurement of phase noise, as it is clarified by observing the fast switching occurring between two specified fringes on an oscilloscope. The effect of phase noise is such that switching times corresponding to successive observations of the same fringes have a randomness, and there is a statistical distribution of switching instants around the most probable. This is shown in Fig.15. Now, the hysteresis in the power–phase characteristic of the self–mixing waveform prevents the occurrence of multiple switching, with opposite sign, for a single period of target oscillation. The phase noise variance, with Gaussian statistics in practice, can be evaluated by acquisition of the fringe switching times, and by subsequent statistical data analysis, yielding a value of the RMS phase noise. Fig.16 reports the measurement of the RMS phase noise as a function of target distance L_0 for different SL types. For small distances, mechanical fluctuations of the set-up may introduce an error, but for longer distance the theoretical linear dependence is obtained. The line-width is estimated from the slope of the fitted curves. From Fig.16 it is deduced that the line-width for the Mitsubishi ML2701 SL (850 nm Fabry-Perot) is 14.4 MHz for 25 mA injection current and

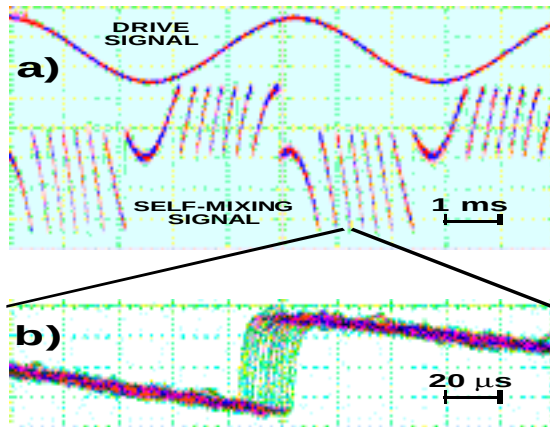


Fig.15. a) Lower trace : sawtooth-like self-mixing signal waveform. Upper trace: loudspeaker drive signal. b) Zoomed frame reporting the superposition of subsequent single-sweep acquisitions of the self-mixing signal corresponding to a specific The switching time randomness can be observed.

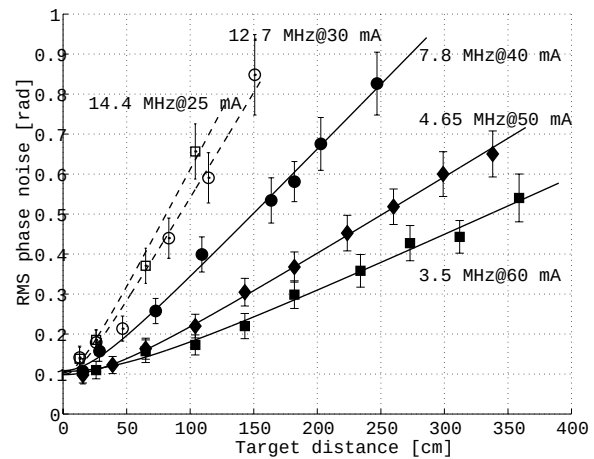


Fig.16. RMS interferometric phase noise as a function of target distance D_0 for different laser diodes and varying pump currents. Empty symbols refer to LD1 (ML2701, 850 nm Fabry-Perot) ; full symbols refer to LD2 (SDL5401, 800 nm Fabry-Perot)

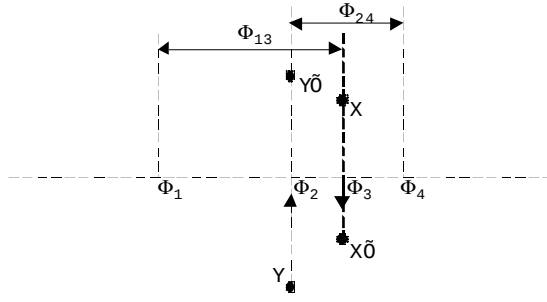


Fig.17. Calculated plot of the function $F(\phi)$ (for $C = 2$ and $\alpha = 3$), with relevant phase segments shown.

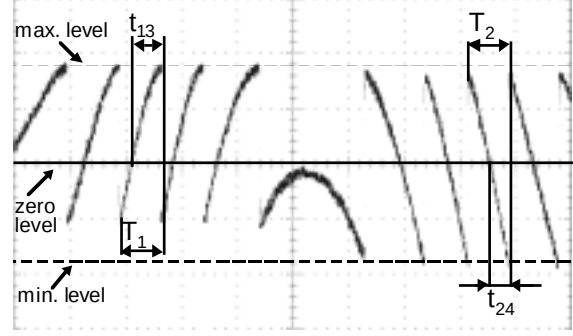


Fig.18. Experimental self-mixing waveform used to determine α .

12.7 MHz for 30 mA. For another SL (SDL SDL5401, 800 nm Fabry-Perot) the line-width is 7.8 MHz for 40 mA injection current, 4.65 MHz for 50 mA, and 3.5 MHz for 60 mA. The line-width values measured using the self-mixing technique show the expected decrease for increasing injection current, and they are in good agreement with the ones obtained by the self-heterodyne method [35] for the same SLs [36].

5.3 Line-width Enhancement Factor

It is well known that SLs exhibit a strong variation of refractive index and gain when the injected carrier density is changed. The parameter describing this dependence is called line-width enhancement factor α [37] and it is defined as: $\alpha = (\partial n_R / \partial N) / (\partial n_I / \partial N)$, where N , n_R , and n_I are respectively the carrier density and the real and imaginary part of the refractive index. The value of α is important in many SL applications, as it characterizes the line-width, the chirp and the response to optical feedback. Using the self-mixing technique, a measurement of α can be obtained, by looking at the dependence of the self-mixing signal on α [38] as predicted by the Lang-Kobayashi theory.

The details are discussed in a companion paper [39], so here we just summarize the results. Of the self-mixing waveform, we measure the phase values ϕ_1 and ϕ_4 corresponding to a zero-crossing of the function $F(\phi)$, and ϕ_2 and ϕ_3 , the phase values corresponding to switches of $F(\phi)$. Analytical expressions are found for $\phi_{13} = \phi_1 - \phi_3$, and $\phi_{24} = \phi_2 - \phi_4$, that is:

$$\phi_{13} = \sqrt{C^2 - 1} + \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(-\frac{1}{C}\right) - \arctan(\alpha) + \frac{\pi}{2} \quad (6a)$$

$$\phi_{24} = \sqrt{C^2 - 1} - \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(-\frac{1}{C}\right) + \arctan(\alpha) - \frac{\pi}{2} \quad (6b)$$

Measuring the above quantities on the self-mixing waveform as shown in Fig.18, and standardizing to T_1 and T_2 , the measured periods of a complete

interferometric fringe, allows us to plot the quantities X_{13} and X_{24} as indicated in Fig.19.

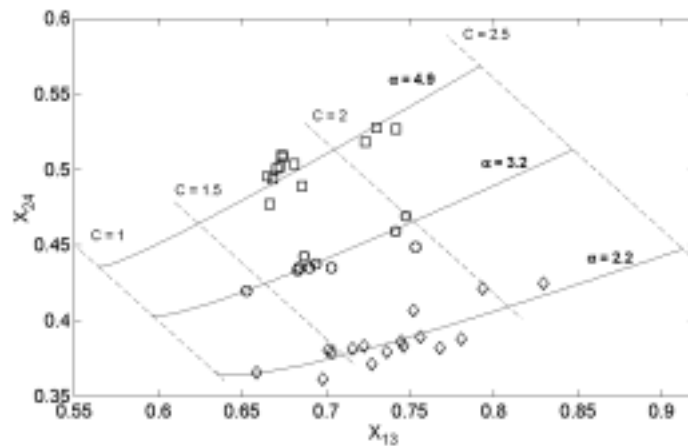


Fig.19. Experimental data points plotted in the X_{24} – X_{13} plane, obtained from repeated measurements for varying optical feedback strength. Squares: Mitsubishi ML925B11F, 1550 nm, DFB; estimated $\alpha = 4.9$. Circles: Hitachi HL8325G#1, 820 nm, Fabry–Perot; estimated $\alpha = 3.2$. Diamonds: SDL SDL-7511-G1, 635 nm, DFB; estimated $\alpha = 2.2$.

Contour lines for constant C values are also plotted.

In Fig.19, measured data is plotted on the X_{24} – X_{13} plane for three different LDs as obtained by varying the optical feedback strength. A good agreement is found with the extrapolated α values, for which theoretical curves are also plotted. Compared to other methods, the new approach has the advantage that it does not require the measurement of the feedback strength, which cannot in general be determined with good accuracy. Moreover, the approach can be also useful for a measure of the effective feedback strength.

6 CONCLUSIONS

Applications of the laser diode self-mixing interferometric method have been reviewed, illustrating the many interesting areas in which this technique can provide a unique opportunity of development of several practical instruments.

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